6[K].—CHARLES W. DUNNETT, "Tables of the bivariate normal distribution with correlation $1/\sqrt{2}$," 1958, 28 cm. Deposited in UMT File.

The bivariate normal probability distribution,

$$L(h, k; r) = \frac{1}{2\pi\sqrt{1 - r^2}} \int_h^\infty \int_k^\infty \exp\left[-\tfrac{1}{2}\left(\frac{x^2 - 2rxy + y^2}{1 - r^2}\right)\right] dx\, dy,$$

has been tabulated by Karl Pearson [1] for $r = -1.0(.05)1.0$. The present tables were prepared to avoid the necessity of interpolating in Pearson's tables when $r = \pm 1/\sqrt{2}$. This case arises in certain double sampling procedures in which probability statements concerning X and $X + Y$ jointly are required, where X and Y are independent normal chance variables with the same variance.

The tables were computed on a Royal McBee LGP-30 electronic computer, by numerical quadrature, using the relation

$$L(h, k; r) = \int_h^\infty \left[1 - F\left(\frac{k - rx}{1 - r^2}\right)\right] f(x)\, dx$$

where $f(x) = (1/\sqrt{2\pi}) \exp(-x^2/2)$ and $F(x) = \int_{-\infty}^x f(x)\, dx$. The function is tabulated for $r = 1/\sqrt{2}$ in Table I and for $r = -1/\sqrt{2}$ in Table II for positive values of its arguments, h varying in steps of 0.1, and k varying in steps of $0.1\sqrt{2}$. All entries are given to six decimals and should be correct to this number of places.

The function can be determined for negative values of its arguments by using the relationships

$$L(-h, k; r) = L(-\infty, k; r) - L(h, k; -r)$$
$$L(h, -k; r) = L(h, -\infty; r) - L(h, k; -r)$$
$$L(-h, -k; r) = L(h, k; r) + 1 - L(h, -\infty; r) - L(-\infty, k; r)$$

where $L(h, -\infty; r) = L(-\infty, h; r) = 1 - F(h)$, which is the right-hand tail area of the univariate normal distribution. In order to avoid the necessity of consulting tables of $F(h)$, these values are included in the table.

Tables III, IV and V were computed from Tables I and II using these relationships. The error in the entries in Tables III and IV should be no greater than a unit in the sixth decimal place. The error in the entries in Table V should be no greater than two units in the sixth decimal place. All tables have been deposited in the Unpublished Mathematical Tables repository.

AUTHOR'S ABSTRACT

1. KARL PEARSON, *Tables for Statisticians and Biometricians*, Part II, Cambridge University Press, 1931.

7[L].—E. A. CHISTOVA, *Tablit͡sy funkt͡siĭ Besseli͡a ot deĭstvitel'nogo argumenta i integralov ot nikh͡* (*Tables of Bessel functions of real argument and of integrals involving them*), Izdatel'stvo Akademiĭ Nauk SSSR, Moscow, 1958, 524 p. + loose card, 28 cm. Price 45 rubles.

This important volume in the now familiar series of Mathematical Tables of the Computational Center of the Academy of Sciences was initiated by V. A. Ditkin.

The main table occupies pages 23–522 and gives values for

$$x = 0(.001)15(.01)100$$

of the eight functions

$$J_n(x), \qquad Ji_n(x) = \int_x^\infty \frac{J_n(u)}{u}\,du,$$

$$Y_n(x), \qquad Yi_n(x) = \int_x^\infty \frac{Y_n(u)}{u}\,du,$$

where $n = 0$ and 1. The values are to 7D or, near singularities at the origin, 7S Auxiliary functions (detailed below) are provided for $x = 0(.001).150$. Over the range $x = 1.350(.001)15$ there are no differences, and linear interpolation provides results correct within two units of the last place. Second differences are required for some functions in parts of the ranges $x = .150(.001)1.350$ and $x = 15(.01)100$; the quantities printed (when greater than about 16) are sums of two consecutive second differences, for use in Bessel's formula.

The values of $J_0(x)$ and $J_1(x)$ are rounded to 7D from the well-known Harvard tables, and are included for convenience; the values of the other six quantities result from original computations, and may be checked only partially against previous, less extensive, tables which are listed in a bibliography. The integrals $Ji_0(x)$ and $Ji_1(x)$ were computed by Simpson's rule on an electronic computer and other machines. The functions $Y_0(x)$, $Y_1(x)$, $Yi_0(x)$, and $Yi_1(x)$ were evaluated on the electronic computer BESM, using Taylor series and asymptotic expansions. All values were checked by differencing. By means of formulas given on pages 11–12, the integrals of $J_0(u)$, $J_1(u)$, $Y_0(u)$, and $Y_1(u)$, *not* divided by u, may be simply expressed in terms of the tabulated functions.

The nine auxiliary functions given on pages 17–19 are all tabulated for $x = 0.(.001).150$ to 7D without differences. The functions are:

$$Li_0(x) = Ji_0(x) + \ln \tfrac{1}{2}x$$

$$C_0(x) = Y_0(x) - (2/\pi)J_0(x)\ln x \qquad E_0(x) = (2/\pi)\{Ji_0(x) + \ln x\}$$

$$C_1(x) = x\{Y_1(x) - (2/\pi)J_1(x)\ln x\} \qquad E_1(x) = (2/\pi)\{Ji_1(x) - 1\}$$

$$D_0(x) = (2/\pi)J_0(x) \qquad F_0(x) = Yi_0(x) - (2/\pi)\ln x\{Ji_0(x) + \tfrac{1}{2}\ln x\}$$

$$D_1(x) = (2/\pi)J_1(x) \qquad F_1(x) = x\{Yi_1(x) + (2/\pi)\ln x[1 - Ji_1(x)]\}$$

It is stated that the errors do not exceed 0.6 final unit, except that they may attain one final unit in the case of the auxiliary functions $C_0(x)$, $C_1(x)$, $F_0(x)$, and $F_1(x)$.

A table of the Bessel coefficient $\frac{1}{4}t(1 - t)$ for $t = 0(.001)1$ to 5D without differences is given on page 523 and also on a loose card.

A. F.

8[L].—Otto Emersleben, "Werte einer Zetafunktion zweiter Ordnung mit Argument $s = 2$," Bearbeitet in der Abteilung Angewandte Mathematik der Universität Greifswald, Greifswald, 1956.