

The main table occupies pages 23–522 and gives values for

$$x = 0(.001)15(.01)100.$$

of the eight functions

$$J_n(x), \quad Ji_n(x) = \int_x^\infty \frac{J_n(u)}{u} du,$$

$$Y_n(x), \quad Yi_n(x) = \int_x^\infty \frac{Y_n(u)}{u} du,$$

where  $n = 0$  and  $1$ . The values are to 7D or, near singularities at the origin, 7S. Auxiliary functions (detailed below) are provided for  $x = 0(.001).150$ . Over the range  $x = 1.350(.001)15$  there are no differences, and linear interpolation provides results correct within two units of the last place. Second differences are required for some functions in parts of the ranges  $x = .150(.001)1.350$  and  $x = 15(.01)100$ ; the quantities printed (when greater than about 16) are sums of two consecutive second differences, for use in Bessel's formula.

The values of  $J_0(x)$  and  $J_1(x)$  are rounded to 7D from the well-known Harvard tables, and are included for convenience; the values of the other six quantities result from original computations, and may be checked only partially against previous, less extensive, tables which are listed in a bibliography. The integrals  $Ji_0(x)$  and  $Ji_1(x)$  were computed by Simpson's rule on an electronic computer and other machines. The functions  $Y_0(x)$ ,  $Y_1(x)$ ,  $Yi_0(x)$ , and  $Yi_1(x)$  were evaluated on the electronic computer BESM, using Taylor series and asymptotic expansions. All values were checked by differencing. By means of formulas given on pages 11–12, the integrals of  $J_0(u)$ ,  $J_1(u)$ ,  $Y_0(u)$ , and  $Y_1(u)$ , not divided by  $u$ , may be simply expressed in terms of the tabulated functions.

The nine auxiliary functions given on pages 17–19 are all tabulated for  $x = 0(.001).150$  to 7D without differences. The functions are:

$$Li_0(x) = Ji_0(x) + \ln \frac{1}{2}x$$

$$C_0(x) = Y_0(x) - (2/\pi)J_0(x) \ln x \quad E_0(x) = (2/\pi)\{Ji_0(x) + \ln x\}$$

$$C_1(x) = x\{Y_1(x) - (2/\pi)J_1(x) \ln x\} \quad E_1(x) = (2/\pi)\{Ji_1(x) - 1\}$$

$$D_0(x) = (2/\pi)J_0(x) \quad F_0(x) = Yi_0(x) - (2/\pi) \ln x\{Ji_0(x) + \frac{1}{2} \ln x\}$$

$$D_1(x) = (2/\pi)J_1(x) \quad F_1(x) = x\{Yi_1(x) + (2/\pi) \ln x[1 - Ji_1(x)]\}$$

It is stated that the errors do not exceed 0.6 final unit, except that they may attain one final unit in the case of the auxiliary functions  $C_0(x)$ ,  $C_1(x)$ ,  $F_0(x)$ , and  $F_1(x)$ .

A table of the Bessel coefficient  $\frac{1}{4}t(1-t)$  for  $t = 0(.001)1$  to 5D without differences is given on page 523 and also on a loose card.

A. F.

8[L].—OTTO EMERSLEBEN, "Werte einer Zetafunktion zweiter Ordnung mit Argument  $s = 2$ ," Bearbeitet in der Abteilung Angewandte Mathematik der Universität Greifswald, Greifswald, 1956.

This paper contains a tabulation of the function

$$Z(x, y) = \sum_{k=-\infty}^{\infty} \sum'_{l=-\infty}^{\infty} \frac{e^{2\pi i(kx+ly)}}{k^2 + l^2}$$

where the prime denotes the fact that the term with  $k = l = 0$  is omitted.

The table is given for values  $x$  and  $y$  in the range

$$\frac{1}{2} \geq x \geq y \geq 0,$$

where  $x$  and  $y = 0(0.01)0.5$ . The entries are given to six and sometimes seven decimal places, and are said to be accurate to at least two units in the last decimal place.

In the calculation of this table, use was made of the seven-place tables of the exponential integrals published in 1954 by the U.S.S.R. Academy of Science, Institute of Exact Mechanics and Computation.

A. H. T.

9[L].—HERMAN H. LOWELL, *Tables of the Bessel-Kelvin Functions Ber, Bei, Ker, Kei, and their Derivatives for the Argument Range 0(0.01)107.50*, Technical Report R-32, National Aeronautics and Space Administration, Washington, D. C., 1959, 300 p., 26 cm.

These tables provide an elaborate and attractively arranged compilation of decimal values of the Bessel-Kelvin functions (frequently referred to simply as the Kelvin functions) of the first and second kinds of order zero, together with their first derivatives. Approximations to ber and bei and their first derivatives appear in floating-point form to generally 13 or 14 significant figures. On the other hand, the number of significant figures given for ker and kei and their first derivatives vary from 9 to 13, according to a pattern explained in the detailed introduction, which also describes the construction of these tables and the checks applied to the tabular entries. The calculations were performed on an IBM 650 calculator using the Bell Telephone Laboratories Double-Precision (16-figure) Interpretive System.

In addition to the checks applied by the author, the reviewer collated the values of ber  $x$  and bei  $x$  with similar data given by Aldis [1] to 21D, for the range  $x = 0.1(0.1)6.0$ . No discrepancies were detected.

The range, precision, and accuracy of the tables under review establish them as the definitive tables of the Kelvin functions at the present time.

J. W. W.

1. W. STEADMAN ALDIS, "On the numerical computation of the functions  $G_0(x)$ ,  $G_1(x)$ , and  $J_n(x\sqrt{i})$ ," Roy. Soc. London, *Proc.*, v. 66, 1900, p. 32-43.

10[L].—NUMERICAL COMPUTATION BUREAU, Report No. 11, *Tables of Whittaker Functions (Wave Functions in Coulomb Field) Part 2*, The Tsuneta Yano Memorial Society, 1-9 Yuraku-cho, Chiyoda-Ku, Tokyo, Japan, 1959, 52 p., 26 cm. Price \$3.00.

The first part of these tables was reviewed in *MTAC*, v. 12, 1958, p. 86-88. The earlier review contains some errors and fails to give complete information.

The functions considered are defined thus:

$$G_{\xi, l} + iF_{\xi, l} = \exp\left(-\frac{x}{2}\xi\right) \exp\left[-i\left(\frac{l}{2}\pi - \sigma_l\right)\right] W_{i\xi, l+1/2}(-ix).$$