

- 5) The column-normal isomorph of the same multiplication table (omitted if the row-normal table is itself column-normal). The semigroup is anti-isomorphic to itself if and only if item 4 is omitted and item 5 is included.

There is a bibliography of 34 titles, including work of a Japanese group including Professor Takayuki Tamura. It is notable that the Japanese group, working by hand, obtained all 126 semigroup types of order 4 prior to SWAC (without error), and that they finished obtaining the semigroups of order 5 almost simultaneously with the American group of Motzkin, Selfridge, and SWAC (but with at least one error discovered by the American group). It appears that in computing semigroups it has been a reasonably fair match between Japanese without an abacus and Americans armed with an electronic digital computer! The text of the dissertation reviewed here does not refer to the Japanese computation for order 5.

Minor criticisms: On page 8, line 8, for "... an element of a semigroup ..." read "... an element  $a$  of a semigroup ...". In the footnote on page 8, for  $m$  read  $a$ . In reference [A2] the authors' names are permuted. In this and other references the authors' first names have been carefully omitted [why is this done so often?].

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1. GEORGE E. FORSYTHE, "SWAC computes 126 distinct semigroups of order 4," *Amer. Math. Soc., Proc.*, v. 6, 1955, p. 443-447.

31[K].—E. L. LEHMANN, *Testing Statistical Hypotheses*, John Wiley & Sons, Inc., New York, 1959, xiii + 369 p., 24 cm. Price \$11.00.

This long-awaited book is a welcome addition to Wiley's fine series of texts in the different areas of modern mathematical statistics, and the author is to be thanked and congratulated for a difficult but needed job exceedingly well done. The title does not convey an adequate idea of the scope of the book, which includes material from many branches of statistics, nor of the exceedingly helpful devices used that make it possible to catch up with the most recent advances. Going through it is a refreshing and stimulating experience.

The book provides "a systematic account of the theory of hypothesis testing and of the closely related theory of estimation by confidence sets. The principal applications of these theories are given, including the one- and two-sample problems concerning normal, binomial and Poisson distributions. There is also a treatment of permutation tests and of some aspects of the analysis of variance and of regression analysis. Introductions to multivariate and sequential analysis, and to non-parametric tests are offered. Methods based on large sample considerations ( $\chi^2$  and likelihood ratio tests) are sketched. The emphasis throughout is on the various optimum properties of the procedures. These are discussed in terms of the Neyman-Pearson formulation, but against a background of decision theory which frequently permits a broader justification of the results."

The level of the treatment is set by the fact that "the natural framework for a systematic treatment of hypothesis testing is the theory of measure in abstract spaces." By unrestricted use of the abstract approach, the author is enabled to bring the prepared reader abreast of the very latest developments. For this, one should have an appreciation, if not a knowledge, of concepts in measure theory,

as well as a considerable background in "classical" mathematical statistics. It should be stated, however, that all necessary theorems are stated and discussed, if not always proved.

The text is greatly enriched by some very valuable time-saving features, such as footnotes that immediately relate an outside source to the point under discussion, and annotated lists of references, appearing at the close of each chapter and frequently summarizing a paper in a single sentence, thereby giving the reader a bird's-eye view of the latest pertinent papers as well as of the earlier literature. Furthermore, for each section a set of substantial exercises, totalling over 200, is provided. Many of these are accompanied with outlines of solutions, and provide introductions to additional topics.

Attention to estimation as a special case of hypothesis testing is essentially limited to confidence sets, point estimation receiving very little consideration. For the sake of completeness one would like to have seen some treatment of the Cramér-Rao inequality and its modern development by Bhattacharya and others, as well as minimum-variance estimation on which so much practical work is based. This, however, is hardly a criticism, since treatment of estimation is not a main purpose.

While not designed as a "cookbook" in the analysis of actual data, because of its advanced nature the book does give a deep understanding of the many tests and their relationships to a unified theory. As such, and in view of its time-saving features, the book is well worth the price, and should be in the possession of the advanced worker in mathematical statistics and others having the requisite background.

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32[L].—L. N. KARMAZINA & É. A. CHISTOVA, *Tablitsy funktsii Besseliâ ot mnimogo argumenta i integralov ot nih* (Tables of Bessel functions of imaginary argument and of integrals involving them). Izdatel'stvo Akademii Nauk SSSR (Press of the Academy of Sciences of the USSR), Moscow, 1958, 328 p., 27 cm. Price 37 rubles 15 kopecks.

This volume in the series of Mathematical Tables from the Computational Center of the Academy of Sciences is a continuation of earlier work [1] on Bessel functions of real argument.

The present tables were prepared on the electronic computer STRELA. In the main table (pages 19-328), the values of the following seven functions are given for  $x = 0(.001)5(.005)15(.01)100$ :

$$e^{-x}I_0(x), \quad e^{-x}I_1(x), \quad e^{-x} \int_0^x I_0(u) du, \\ e^x K_0(x), \quad e^x K_1(x), \quad e^x \int_x^\infty K_0(u) du, \quad \text{and} \quad e^x.$$

The values are given here to 7D except near the origin, where they are to 7S; no differences are given. Near the origin, the following auxiliary functions are given: