

$$I_0(x) \quad \text{and} \quad E_0(x) = K_0(x) + \ln x I_0(x)$$

for  $x = 0(.001).15$  to 7S and 7D, respectively;

$$I_1(x) \quad \text{and} \quad E_1(x) = x[K_1(x) - \ln x I_1(x)]$$

for  $x = 0(.001).2$  to 7D. By means of the formulas given on page 11, a number of related integrals can be evaluated by using the present tables.

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1. É. A. Čhistova, *Tablitsy funktsii Besseliâ ot detstvitel'nogo argumenta i integralov ot nih* (Tables of Bessel functions of real argument and of integrals involving them). Izdatel'stvo Akademii Nauk SSSR (Press of the Academy of Sciences of the USSR), Moscow, 1958. *Math. Comp.*, v. 14, 1960, p. 79-80.

33[L].—F. A. PAXTON & J. E. ROLLIN, *Tables of the Incomplete Elliptic Integrals of the First and Third Kind*, Curtiss-Wright Corporation, Research Division, Quehanna, Pennsylvania, June 1959, 436 p., 28 cm.

This large table gives values of the elliptic integral of the third kind, which in the notation of the authors is

$$\Pi(\phi, \alpha^2, k) = \int_0^\phi \frac{d\theta}{(1 - \alpha^2 \sin^2 \theta) \sqrt{1 - k^2 \sin^2 \theta}}$$

including, as a special case, the elliptic integral of the first kind,  $F(\phi, k) = \Pi(\phi, 0, k)$ . Values are tabulated to 7D without differences for  $\phi = 0(1^\circ)90^\circ$ ,  $\alpha^2 = 0(.02)1$ ,  $k^2 = 0(.02)1$ . The values were computed on an IBM 704 by Simpson's rule. The authors appear to claim general accuracy within 10 final units, except possibly for  $\sin \phi$ ,  $\alpha^2$ , and  $k^2$  near unity. The reviewer has encountered nothing which invalidates the general claim, but the exception is certainly to be noticed.

There are several ways in which certain values, especially of the complete integrals ( $\phi = 90^\circ$ ), may be checked from existing tables with little or no arithmetic. Using for brevity the references of *MTAC*, v. 3, 1948, p. 250, let us consider four checks:

(i) Values of  $F(90^\circ, k) = K$  with argument  $k^2$  are given to 10-12D in Hayashi 1. They show that the values of Paxton and Rollin are systematically too small; the error rises from 4 final units at  $k^2 = .02$  to 11 final units at  $k^2 = .98$ . These errors are practically within the claimed limits. The machine value at  $k^2 = 0$ , which should equal  $\frac{1}{2}\pi$ , is 6 final units too small, but has been corrected by hand.

(ii) Values of  $F(\phi, k)$  for  $\phi = 0(1^\circ)90^\circ$ ,  $k^2 = \frac{1}{2}$  (modular angle =  $45^\circ$ ) are given to 10D in Legendre 3, 5, 6, 7, 8 (also to 12D in Legendre 3, 5). They show no errors in Paxton and Rollin exceeding 5 final units.

(iii) Values of

$$\Pi(\phi, 0, 1) = \int_0^\phi \sec \phi \, d\phi,$$

the inverse gudermannian, are given to 9D in Legendre 3, 5, 6, 7, 8 (also to 12D in Legendre 3, 5). They show that the later values of Paxton and Rollin are systematically too small, for example by about 1, 11, 45, 141 final units at  $\phi = 45^\circ$ ,

85°, 88°, 89°, respectively. It may be added that values of  $F(\phi, k)$  for  $\phi = 0(1^\circ)90^\circ$ ,  $k^2 = 0(.01)1$  are given in Samoilova-ĭakhontova 1, but to only 5D.

(iv) When  $\alpha^2 = k^2$ , we have

$$\Pi(90^\circ, \alpha^2, k) = \int_0^{\pi/2} (1 - k^2 \sin^2 \theta)^{-3/2} d\theta,$$

which is known to equal  $E/(1 - k^2)$ , where  $E$  is the complete elliptic integral of the second kind, given to 10D with argument  $k^2$  in Hayashi 3. Evaluation shows that the values of Paxton and Rollin are systematically too small, for example by 4, 15, 31, 177, and almost 590 units at  $k^2 = .2, .8, .9, .96, .98$ , respectively.

As far as the reviewer's examination has gone, it seems likely that the table is correct everywhere to about 4D, almost everywhere to 5D, and in large regions to 6D and 7D. Even with this limitation, the table (which, as far as the reviewer is aware, is a corporation research report rather than a published work) must be regarded as epoch-making in the history of the tabulation of the elliptic integral of the third kind. It should be added that another sizable table of this integral, by Selfridge and Maxfield [1], appeared in 1958, but with a different argument system.

A. F.

1. R. G. SELFRIDGE & J. E. MAXFIELD, *A Table of the Incomplete Elliptic Integral of the Third Kind*, Dover Publications, New York, 1958 (also Constable, London).

**34[P, W].**—ARMOUR RESEARCH FOUNDATION, *Proceedings of the Fourth Annual Computer Applications Symposium*, 1957, sponsored by the Armour Research Foundation of Illinois Institute of Technology, 1958, x + 126 p., 23 cm. Price \$3.00.

This is a collection of 15 papers based on 12 talks, two luncheon addresses and a panel discussion. Practically all the symposium concerned itself with applications of digital computers.

The program of the symposium covered two days, one devoted to a session on "Business and Management Applications," the other to a session on "Engineering and Research Applications."

The following seven papers concerned with the first subject appear in the *Proceedings*:

An Extensive Hospital and Surgical Insurance Record-Keeping System—R. J. KOCH,

A Central Computer Installation as a Part of an Air-Line Reservations System—R. A. McAVOY,

Fitting a Computer into an Inventory-Control Problem—O. A. KRAL,  
The Problems of Planning New Metropolitan Transportation Facilities and Some Computer Applications—J. D. CARROLL, JR.,

Data-Processing Tasks for the 1960 Census—D. H. HEISER & DOROTHY P. ARMSTRONG,

The Handling of Retail Requisitions from a General Warehouse—M. J. STOUGHTON,

Automatic Programming for Business Applications—GRACE M. HOPPER.