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E = total energy, $2\pi\hbar = \text{Planck's constant},$

a = radius of the well.

Using this table, the first few roots have been obtained graphically and are recorded in Table 1 to three significant digits. For most practical purposes, these values should be satisfactory. If necessary, they can be improved by use of Newton's method.

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1. M. Onoe, Tables of Modified Quotients of Bessel Functions of the First Kind for Real and Imaginary Arguments, Columbia University Press, New York, 1958.

A Note on Factors of $n^4 + 1$

By A. Gloden

The factorizations enumerated in this note form a sequel to my published factor table [1] of integers $n^4 + 1$. They have been obtained by means of my table of solutions of the congruence $x^4 + 1 \equiv 0 \pmod{p}$ for primes lying between $8 \cdot 10^5$ and 10^6 [2].

The following numbers are primes:

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n^4 + 1 for n = 912, 914, 928, 930, 936, 952, 962, 966, 986, 992, 996.
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$$\frac{1}{2}(n^4+1)$$
 for $n=1071, 1087, 1101, 1119, 1123, 1125, 1135, 1163, 1173, 1183.$

$$\frac{1}{17}(n^4+1)$$
 for $n=1562, 1726, 1732, 1834.$

$$\frac{1}{41}(n^4+1)$$
 for $n=1818, 1848, 1982, 2006, 2012, 2064, 2088, 2094, 2228, 2340, 2364.$

$$\frac{1}{73}(n^4+1)$$
 for $n=2346$.

$$\frac{1}{89}(n^4+1)$$
 for $n=2262, 2302, 2544, 2682.$

$$\frac{1}{113}(n^4+1)$$
 for $n=2468$.

$$\frac{1}{187}(n^4+1)$$
 for $n=2476$.

$$\frac{1}{233}(n^4+1)$$
 for $n=2808$.

$$\frac{1}{2\cdot 17}(n^4+1)$$
 for $n=1709,1715,1759,1787,1827,1845,1855,1879,1895,1963,2015,2021,2031,2093,2185,2229,2259,2287,2303,2327,2331.$

$$\frac{1}{2\cdot 4\cdot 1}(n^4+1)$$
 for $n=2211, 2299, 2651, 2761, 2791, 2815.$

$$\frac{1}{2\cdot73}(n^4+1)$$
 for $n=2533, 2577, 2691, 2723, 2857.$

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$$\frac{1}{2\cdot89}(n^4+1)$$
 for $n=2747, 2771, 2885.$
 $\frac{1}{2\cdot97}(n^4+1)$ for $n=2669, 2683, 2749.$

New factorizations are as follows:

$$938^{4} + 1 = 809273 \cdot 956569$$

$$1060^{4} + 1 = 847577 \cdot 1489513$$

$$1348^{4} + 1 = 940169 \cdot 3511993$$

$$1512^{4} + 1 = 926617 \cdot 5640361$$

$$1874^{4} + 1 = 914561 \cdot 13485457$$

$$2100^{4} + 1 = 17 \cdot 873553 \cdot 1309601$$

$$2838^{4} + 1 = 868841 \cdot 74663657$$

$$2908^{4} + 1 = 41 \cdot 940369 \cdot 1854793$$

$$\frac{1}{2}(1155^{4} + 1) = 830233 \cdot 1071761$$

$$\frac{1}{2}(1191^{4} + 1) = 935353 \cdot 1075577$$

$$\frac{1}{2}(2635^{4} + 1) = 872369 \cdot 2971849$$

$$\frac{1}{2}(2765^{4} + 1) = 857569 \cdot 28107577$$

$$\frac{1}{2}(2765^{4} + 1) = 908353 \cdot 32173321$$

$$\frac{1}{2}(2977^{4} + 1) = 17 \cdot 809041 \cdot 2855393$$

The following factorization was omitted from my original table [1]:

$$\frac{1}{2}(2055^4 + 1) = 17.572233.916633.$$

The least integers still incompletely factored correspond to n = 1038 and 1229, for even and odd values of n, respectively.

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- 1. A. Gloden, "Table de factorisation des nombres $n^4 + 1$ dans l'intervalle 1000 < n < 3000," Institut Grand-Ducal de Luxembourg, Archives, Tome XVI, Luxembourg, 1946, p. 71-88.
- 2. A. Gloden, Table des Solutions de la Congruence $x^4 + 1 \equiv 0 \pmod{p}$ pour 800,000 , published by the author, rue Jean Jaurès, 11, Luxembourg, 1959.

A Note on the Solution of Quartic Equations

By Herbert E. Salzer

For any quartic equation with real coefficients,

(1)
$$X^4 + AX^3 + BX^2 + CX + D = 0,$$

the following condensation of the customary algebraic solution is recommended as quickest and easiest for the computer to follow (no mental effort required). It works in every exceptional case.

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