

minimum variance, is the other estimate. Table II compares variances and efficiencies of these two estimates to  $3S$  for  $N = 2(1)20$ . Also a column gives the ratio of the variance of (BLSS) as given in [2] to the variance of  $S'$  to  $3D$  for  $N = 2(1)20$ .

J. R. VATNSDAL

State College of Washington  
Pullman, Washington

1. A. K. GUPTA, "Estimation of the mean and standard deviation of a normal population for a censored sample", *Biometrika*, v. 39, 1952, p. 260-273.

2. A. E. SARHAN & B. G. GREENBERG, "Estimation of location and scale parameters by order statistics from singly and doubly censored samples. Part I", *Ann. Math. Stat.* v. 27, 1956, p. 427-451.

49[K].—H. F. DODGE & H. G. ROMIG, *Sampling Inspection Tables*, Second Edition, John Wiley & Sons, Inc., New York, 1959, xi + 224 p., 29 cm. Price \$8.00.

The first feature one notices about the second edition of the Dodge-Romig tables, as compared to the first edition, is its size. Measuring  $8\frac{1}{2}'' \times 11''$ , with a total of 224 pages, it stands in bold contrast to the pocket-sized  $5\frac{1}{2}'' \times 8\frac{1}{2}''$ , 106-page first edition,—a four-fold increase, in a sense indicative of the growth of statistical quality control since the first edition was published in 1944.

There are 60 pages of text covering an introduction, and four chapters which describe the principles, procedure for application, and the mathematics of the sampling plans. The titles of these chapters are, respectively: A Method of Sampling Inspection; Single Sampling and Double Sampling Inspection Tables; Using Double Sampling Inspection in a Manufacturing Plant; and Operating Characteristics of Sampling Plans. The remaining 158 pages include a table of contents, seven appendices, and an index. The last four appendices give the same four sets of tables as appear in the first edition; these are respectively entitled: Single Sampling Tables for Stated Values of Lot Tolerance Per Cent Defective (LTPD) with Consumer's Risk of 0.10; Double Sampling Tables for Stated Values of Lot Tolerance Per Cent Defective (LTPD) with Consumer's Risk of 0.10; Single Sampling Tables for Stated Values of Average Outgoing Quality Limit (AOQL); Double Sampling Tables for Stated Values of Average Outgoing Quality Limit (AOQL). The first three appendices are devoted to 120 pages of operating characteristic curves; these are, respectively, OC Curves for all Single Sampling Plans in Appendix 6; OC Curves for all Double Sampling Plans in Appendix 7; and OC Curves for Single Sampling Plans with  $c = 0, 1, 2, 3$ , and  $n \leq 500$  (based on binomial probabilities).

As can be seen from the above list of contents, the largest part of the increase in size is due to the inclusion of three sets of operating characteristic curves,—two sets for the AOQL plans, and one set for a separate inventory of single sampling plans. This separate inventory has a wide enough range in sample size and acceptance numbers to provide a useful independent reference of OC curves for those who prefer to derive their own single sampling plans (charts to derive such plans have been retained from the first edition). Although separate sets of OC curves are not given for the LTPD plans, the authors point out how these may be obtained or estimated from the OC curves which are given.

It is heartening to see this inclusion of OC curves and the authors' comment

that it "has been urged over the years by a number of engineers", as well as the inclusion of a completely new chapter devoted to a discussion of the operating characteristic curves. This overt endorsement by this distinguished team in the field of quality control of the use of the OC curve to evaluate an acceptance sampling plan should impress upon quality control practitioners the importance of describing the assurance provided by a decision-making procedure in probability terms. Perhaps it will also emphasize the fact that while one may prefer to attach the label of LTPD plan, AOQL plan, or AQL plan to an acceptance sampling plan, depending on first considerations in deriving or classifying a plan, these sampling plans are one and the same as far as assurance in decision making is concerned, if they have the same operating characteristic curve.

The clear distinction made by the authors between OC curves giving the probability of lot acceptance based on lot quality as distinguished from process quality is most welcome, as this distinction is seldom clearly made. That there is a difference is often not realized; sometimes it is misunderstood and the importance of the difference exaggerated; at best it is ignored, since most often, but not always, the difference in OC curves is slight, as the authors point out. It is, however, unfortunate, in this reviewer's opinion, that the authors chose to attach the special labels of Type A and Type B to the corresponding OC curves, rather than simply to identify them as finite lot quality and infinite lot or process quality OC curves. The additional labels are not essential, and can do nothing more than add mystery to an already confused situation to the many who will not look beneath the labels. Unfortunately, also, an inaccuracy has crept into a statement with regard to these OC curves. On page 59 starting at the bottom of the first column the authors state, "When the sample sizes are a larger percentage of the lot size, the Type A OC curve will fall somewhat below the Type B curve shown on the chart, as can be seen in Fig. 4-1 where the Type A OC Curve for  $N = \infty$  is identically the Type B OC curve." That the statement is inaccurate can indeed be seen from a careful examination of Fig. 4-1, since the finite lot (Type A) and infinite lot (Type B) OC curves intersect and cross. It would be better to remember that for the same sampling plan the OC curve for a finite lot is always more discriminating than the OC curve for an infinite lot.

Little need be said about the tables of sampling plans, which are well known as a result of the first edition. Derived on the principle of minimizing the total amount of inspection, sampling as well as screening of rejected lots, they are particularly suited to producers who are responsible for both the production and sampling inspection of their finished product. The sampling plans may also be used by a purchaser for acceptance inspection, but the choice of plan for this purpose should be based principally on the properties of the operating characteristic curve.

The format and typesetting, including tables and graphs, are considerably improved over the first edition. The division of each page of text into two columns also improves on readability. A minor typographical error which occurs in the second edition, but not in the first, appears on page 33, equation (2-1a), where  $C_N^M$  should be  $C_m^M$ .

The book can be highly recommended to those with modest or little mathematical background. The improvements in the second edition are sufficient to warrant its own place, along with other worthy texts, on the bookshelf of students and

practitioners of quality control who are interested in a comprehensive account of sampling inspection as well as in the procedures and tables for its application.

HARRY M. ROSENBLATT

Federal Aviation Agency  
Washington, District of Columbia

**50[K].**—C. W. DUNNETT & R. A. LAMM, "Some tables of the multivariate normal probability integral with correlation coefficients  $\frac{1}{3}$ ," Lederle Laboratories, Pearl River, New York. Deposited in UMT File.

The probability integral of the multivariate normal distribution in  $n$  dimensions, having all correlation coefficients equal to  $\rho$  (where necessarily  $-\frac{1}{n-1} < \rho < 1$ ), is given by

$$\int_{-\infty}^{x_1} \cdots \int_{-\infty}^{x_m} \left(\frac{1}{2\pi}\right)^{n/2} \frac{[1 + (n-1)\rho]^{-1/2}}{(1-\rho) \frac{(n-1)}{2}} \exp \left[ \frac{1 + (n-2)\rho}{(1-\rho)[1 + (n-1)\rho]} \right. \\ \left. \cdot \left\{ \sum x_i^2 - \frac{2\rho}{1 + (n-2)\rho} \sum_{i \neq j} \sum x_i x_j \right\} \right] dx_1 \cdots dx_n$$

This function, which we shall denote by  $F_{n,\rho}(x_1, \dots, x_n)$ , has been tabulated for  $\rho = \frac{1}{2}$  and  $x_1 = \dots = x_n$  by Teichroew [1]. In the present paper, we present a table for the case  $\rho = \frac{1}{3}$  and  $x_1 = \dots = x_n$ . The need for this table arose in connection with a multiple-decision problem considered by one of the authors [2].

In computing the table, use was made of the fact that, for  $\rho \geq 0$ ,  $F_{n,\rho}(x_1, \dots, x_n)$  belongs to a class of multivariate normal probability integrals which can be written as single integrals (see Dunnett and Sobel [3]), a fact which greatly facilitates their numerical computation. In this case, we have

$$F_{n,\rho}(x_1, \dots, x_n) \equiv \int_{-\infty}^{+\infty} \prod_{i=1}^n \left[ F \left( \frac{x_i + \sqrt{\rho}y}{\sqrt{1-\rho}} \right) \right] f(y) dy$$

where

$$f(y) = \frac{1}{\sqrt{2\pi}} e^{-y^2} \quad \text{and} \quad F(y) = \int_{-\infty}^y f(y) dy.$$

The attached table was computed by replacing the right-hand side of this identity by the series based on the roots of Hermite polynomials described by Salzer *et al.* [4]. Those tabular values marked with an asterisk have been checked by comparison with the values obtained by applying Simpson's rule. The values checked were found to be systematically less than the Simpson's rule values by an amount which varied between .000000 and .000013, depending on  $n$ . This indicates that the error in the tabular values may be no more than 1 or 2 units in 6th decimal place, but further checks are required in order to substantiate this.

The table gives  $F_{n,1/3}(x, \dots, x)$  to six decimal places, with  $x$  varying from 0 to  $7.0/\sqrt{3}$  in steps of  $0.1/\sqrt{3}$  for  $n = 1$  (1) 10, and from  $1.5/\sqrt{3}$  to  $2.1/\sqrt{3}$  in steps of  $0.01/\sqrt{3}$  for  $n = 1$  (1) 10, 13, 18.

AUTHORS' ABSTRACT