where $\bar{x}_1$ and $\bar{x}_2$ are the sample means, and w_1 and w_2 are the sample ranges. Since unequal sample sizes are now permitted, the mean range $(w_1 + w_2)/2$ proposed by Lord [1] in his original paper is no longer used. Moore shows that there is very little loss in power resulting from the use of the simple sum $w_1 + w_2$, rather than a weighted sum of sample ranges, although for the estimation of σ separately, he gives a table of $f(n_1, n_2)$ and $d_{n_1} + fd_{n_2}$ to $3D$ for $n_1, n_2 = 2(1)20$ to estimate σ from

$$g = \frac{w_1 + fw_2}{d_{n_1} + fd_{n_2}},$$

which minimizes the coefficient of variation of the range estimates of population standard deviation.

The main use of this paper is, of course, the tables of percentage points (10%, 5%, 2%, and 1% points) to $3D$ for the statistic u, above. The tables of percentage points were computed by making use of Patnaik's chi-approximation for the distribution of the range, which resulted in sufficient accuracy. The limits for sample sizes are $n_1, n_2 = 2(1)20$, which fulfills the most usual needs in practice. With this work of Moore, therefore, the practicing statistician has available a very quick and suitably efficient procedure for testing the hypothesis of equal means for two normal populations of equal variance.

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1. E. Lord, "The use of range in place of standard deviation in the t-test", *Biometrika*, v. 34, 1947, p. 41–67.

55[K].—National Bureau of Standards, *Tables of the Bivariate Normal Distribution Function and Related Functions*, Applied Mathematics Series, No. 50, 1959, xliii + 258 p., 27cm. U. S. Government Printing Office, Washington, D. C. Price $3.25.

These tables, compiled and edited by the National Bureau of Standards, provide values for the probability content $L(h, k, r)$ of an infinite rectangle with vertex at the cut-off point (h, k) under a standardized and centered bivariate distribution with correlation coefficient r:

$$L(h, k, r) = \frac{1}{2\pi\sqrt{1 - r^2}} \int_h^\infty \int_k^\infty \exp\left[-\frac{1}{2}\left(x^2 + y^2 - 2rxy\right)/(1 - r^2) \right] dx\, dy$$

The range of tabulation is $h, k = 0(.1)4$, $r = 0(.05)0.95(.01)1$, the values of $L(h, k, r)$ being given to 6 decimal places. For negative correlations, the range of tabulation is $h, k = 0(.1)h_n, k_n$, $r = 0(.05)0.95(.01)1$, the values of $L(h, k, r)$ being given to 7 decimal places, where $L(h_n, k_n, -r) \leqq \frac{1}{2}.10^{-7}$ if h_n and k_n are both less than 4. The two tables of $L(h, k, r)$ for positive and negative r, respectively (Tables I and II in the text), may therefore be regarded as extensions of Karl Pearson's tables of the bivariate normal distribution in his celebrated *Tables for Statisticians and Biometricians*, Part II, since the range of parameters in the latter tables is $h, k = 0(.1)2.6$, $r = -1(.05)1$. In this connection, the authors of the present tables present a list of 31 errors in Pearson's tables, together with the corresponding correct values.

Table III gives the values to 7 decimal places, with the last place uncertain by 2 units, of the probability content, $V(h, \lambda h)$, of a certain right-angled triangle under a centered circular normal distribution with unit variance in any direction. The triangle has one vertex at the center of the distribution, with the angle at that vertex arc tan λ, while the lengths of the two bounding sides at the vertex are h and $h\sqrt{1 + \lambda^2}$. Formally,

$$V(h, \lambda h) = \frac{1}{2\pi} \int_0^h dx \int_0^{\lambda x} \exp\left[-\frac{1}{2}(x^2 + y^2)\right] dy.$$

The range of tabulated values for $V(h, \lambda h)$ is $h = 0(.01)4(.02)4.6(.1)5.6$ and ∞, and $\lambda = 0.1(.1)1$. Table IV gives the values of

$$V(\lambda h, h) = \frac{1}{2\pi} \int_0^{\lambda h} dx \int_0^{x/\lambda} \exp\left[-\frac{1}{2}(x^2 + y^2)\right] dy$$

for the parameters $h = 0(.01)4(.02)5.6$ and ∞, and $\lambda = 0.1(.1)1$, the degree of accuracy being the same as for the values of $V(h, \lambda h)$ in Table III.

Finally, a short table (Table V) for values of $y = \text{arc sin } r/2\pi$, $r = 0(.01)1$, correct to 8 decimal places is provided.

The two-parameter function V is related to the three-parameter function L by the formula

$$L(h, k, r) = V\left(h, \frac{k - rh}{\sqrt{1 - r^2}}\right) + V\left(k, \frac{h - rk}{\sqrt{1 - r^2}}\right) + F,$$

where

$$F = \tfrac{1}{4}[1 - \alpha(h) - \alpha(k)] + y$$

and

$$\alpha(x) = \frac{1}{\sqrt{2\pi}} \int_{-x}^{x} \exp\left(-\frac{1}{2}t^2\right) dt.$$

This relationship enables $L(h, k, r)$ to be computed to 6-decimal accuracy from the 7-decimal values of $V(h, \lambda h)$ in regions where interpolation in $L(h, k, r)$ is difficult.

The function V is also of considerable intrinsic interest, and finds applications in such fields as the probability content of polygons when the underlying distribution is bivariate normal, the distribution of range for normal samples of size 3, the non-central t-distribution for odd degrees of freedom, and one-dimensional heat flow problems. These applications are illustrated clearly and in detail in the section of the book headed Application of the Tables, due to Dr. D. B. Owen. The latter section also provides illustrations of the use of $L(h, k, r)$ in problems ranging from measurement errors, calibration systems, double-sample test procedures, and percentage changes in sample means to some interesting problems in selection (involving the correlation between aptitude test and job performance) and estimation of correlation. Finally, an introductory section discusses the mathematical properties of the L- and V-functions as well as methods of interpolation in the tables.

The authors are to be heartily commended for this most useful book, which will place many statisticians, both practising and those more theoretically inclined, in their debt. The visual appearance and general presentation of the material are excellent. Perhaps one very minor flaw is that since $L(h, k, r) = L(k, h, r)$, tables of $L(h, k, r)$ for $h \geqq k$ would have been sufficient. However, this is a flaw (if at all) from the point of view of economics, but hardly so from the point of view of the user of the tables! The cost of the book is remarkably low.

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56[K].—E. NIEVERGELT, "Die Rangkorrelation U," *Mitteilungsblatt für Math. Stat.*, v. 9, 1957, p. 196–232.

In contrast to Spearman's rank correlation R and Kendall's coefficient T, the author studies the van de Waerden coefficient U. Let p_i and q_i $(i = 1, 2, 3 \cdots n)$ be the ranks of n observations on two variables x and y, and let ξ_i, η_i, ζ_i be the inverses of the normal probabilities: $F(\xi_i) = p_i/(n + 1)$; $F(\eta_i) = q_i/(n + 1)$; $F(\zeta_i) = i/(n + 1)$. Then, U is defined by $U = \sum_{i=1}^{n} \xi_i\eta_i / \sum_{i=1}^{n} \zeta_i^2$.

If x and y are independent the expectation of U is zero and its standard deviation is $\sigma_U = (n - 1)^{-1/2}$, as for Spearman's coefficient. The author calculates also the 4th and 6th moments of U and R, which differ. For n large, U is asymptotically normally distributed about mean zero with standard deviation σ_U. The distribution of U (to $4D$) is tabulated completely for $n = 4$, and over the upper 5% tail for $n = 5, 6, 7$. For larger values of n the Gram-Charlier development is used. Tables testing independence based on 5%, 2.5%, 1%, and .5% probabilities are given to $3D$ for $n = 6(1)\ 30$. For $n > 30$ the normal probability function can be used.

In the case of dependence the correlation between R and U decreases slowly with n increasing. If x and y are normally distributed with zero mean, unit standard deviation and correlation ρ a generalization U^* of U to the continuous case leads to $U^* = \rho$. A consistent estimate for ρ is given. The U test is more powerful than the R test.

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57[K].—D. B. OWEN & D. T. MONK, *Tables of the Normal Probability Integral*, Sandia Corporation Technical Memorandum 64-57-51, 1957, 58 p., 22 x 28 cm. Available from the Office of Technical Services, Dept. of Commerce, Washington 25, D. C., (Physics (TID-4500, 13th Edn.), Price \$.40.

The following forms of the normal probability integral

$$G(h) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{h} e^{-t^2/2}\, dt, \; h \geqq 0$$

$$G(-h) = 1 - G(h)$$

are given for $h = 0(.001)\ 4(.01)\ 7$, to $8D$. For those having frequent use of $G(h)$ these tables eliminate the simple yet troublesome computation necessary when