

2. E. S. PEARSON, H. O. HARTLEY, *Biometrika Tables for Statisticians*, v. 1, Cambridge University Press, London, 1954, p. 104-111.

3. R. D. GORDON, "Values of Mill's ratio of area to bounding ordinate of the normal probability integral for large values of the argument," *Ann. Math. Stat.*, v. 12, 1941, p. 364-366.

58[K].—A. E. SARHAN & B. G. GREENBERG, "Tables for best linear estimates by order statistics of parameters of single exponential distributions from singly and doubly censored samples," *Amer. Stat. Assn., Jn.*, v. 52, 1957, p. 58-87.

Tables are provided for the exact coefficients of the best linear systematic statistics for estimating the scale parameter of a one-parameter single exponential distribution and the scale and location parameters of a two-parameter single exponential distribution. All possible combinations of samples of size n with the r_1 lowest and r_2 highest values censored are considered for $n \leq 10$. Exact coefficients for the best linear systematic statistic for estimating the mean (equal to the location parameter plus the scale parameter) are also given for the two parameter case. Other tables give the variances, exact or to $7D$, of the estimates obtained and the efficiency relative to the best linear estimate to $4D$ based on the complete sample. These extensive tables are of immediate practical importance in many fields, such as life testing and biological experimentation.

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59[K].—Y. S. SATHE & A. R. KAMAT, "Approximations to the distributions of some measures of dispersion based on successive differences," *Biometrika*, v. 44, 1957, p. 349-359.

Let x_1, \dots, x_n be a random sample from a normal population with variance σ^2 and let

$$\delta^2 = \frac{1}{n-1} \sum_{i=1}^{n-1} (x_i - x_{i+1})^2, \quad d = \frac{1}{n-1} \sum_{i=1}^{n-1} |x_i - x_{i+1}|,$$

$$\delta_2^2 = \frac{1}{n-2} \sum_{i=1}^{n-2} (x_i - 2x_{i+1} + x_{i+2})^2, \quad d_2 = \frac{1}{n-2} \sum_{i=1}^{n-2} |x_i - 2x_{i+1} + x_{i+2}|.$$

The problem is to develop approximations to the distributions of these four types of statistics. Let u be any one of these statistics. The method followed is to assume that u is approximately distributed as $(\chi_\nu^2/c)^\alpha$, where χ_ν^2 has a chi-square distribution with ν degrees of freedom; that is, taking $\lambda = 1/\alpha$, that cu^λ is approximately distributed as χ^2 with ν degrees of freedom. The constants c , α (or λ), and ν are then determined by equating the first three moments of u to those of $(\chi_\nu^2/c)^\alpha$. The results show that a fixed value can be used for α (or λ) if $n \geq 5$. This allows two independent measures of variability u_1 and u_2 , based on the same type of statistic, to be compared by use of the F test when $n \geq 5$ for both statistics. The basic results of the paper are given in Table 1. There, for each of δ^2/σ^2 , d/σ , δ_2^2/σ^2 , and d_2/σ , fixed values are stated for λ , while $3D$ values for ν and $4D$ values for $\log_{10} c$ are given for $n = 5(1) 20, 25, 30, 40, 50$. Table 2 deals with an example. Table 3 lists the results of some approximations to δ^2/σ^2 by $(\chi_\nu^2/c)^\alpha$ for $n = 5, 10, 20, 30, 50$. Table 4 lists for comparison purposes, the upper and lower 1% and 5% points for four

approximations to δ^2/σ^2 when $n = 15, 20$. Table 5 is important; it contains $2D$ values of upper and lower 0.5%, 1.0%, 2.5%, and 5% points for the approximate distribution developed for δ^2/σ^2 . Table 6 lists the results of some approximations to d/σ by $(\chi_v^2/c)^\alpha$ for $n = 5, 10, 20, 30, 50$. Finally, Table 7 furnishes $4D$ values of the β_1, β_2 differences that result from using a fixed λ for the $(\chi_v^2/c)^\alpha$ approximation to the distribution of δ_2^2/σ^2 , and from using a fixed λ for the $(\chi_v^2/c)^\alpha$ approximation to the distribution of d_2/σ , for $n = 5, 7, 10(5)30, 40, 50$.

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60[K].—C. C. SEKAR, S. P. AGARWALA & P. N. CHAKRABORTY, "On the power function of a test of significance for the difference between two proportions," *Sankhya*, v. 15, 1955, p. 381-390.

The authors determine the power function of the following statistical test: a sample of size n is drawn from each of two binomial distributions with unspecified probabilities of success p_1 and p_2 , respectively. The null hypothesis is $H_0: p_1 = p_2 = p$. For the two-sided test (alternative hypothesis: $p_1 < p_2$ or $p_1 > p_2$) at significance level α , the critical region is determined by the following conditions:

1) For a given total number r of successes in the two samples, the conditional probability of rejection under H_0 is $\leq \alpha$.

2) If the partition $(a, r - a)$ of r successes is contained in the critical region and $0 < a < r - a$, then the partition $(a - 1, r - a + 1)$ is contained in the critical region.

3) If the partition $(a, r - a)$ is contained in the critical region, the partition $(r - a, a)$ is contained in the critical region.

A similar definition is used for the one-sided test of H_0 against the alternative $p_1 > p_2$. The critical region is determined using the exact conditional probabilities for these partitions given by S. Swaroop, [1].

The power function for the two-sided test is given to $5D$ for p_1 and $p_2 = .1(.1).9$; $n = 5(5)20(10)50, 100, 200$, and for $a = .05$. For the one-sided test the power function to $5D$ is given for the same levels of p_1, p_2 and n , and for $\alpha = .025$.

The critical region used by the authors is the one defined for the exact test by E. S. Pearson, [2]. However, for small sample sizes the power differs considerably from Patnaik's determinations, which are based on an approximately derived critical region and which use a normal distribution approximation of the probabilities.

Examples of the use of the tables are included.

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1. SATYA SWAROOP, "Tables of the exact values of probabilities for testing the significance of differences between proportions based on pairs of small samples," *Sankhya*, v. 4, 1938, p. 73-84.

2. E. S. PEARSON, "The choice of statistical tests illustrated on the interpretation of data classed in a 2×2 table," *Biometrika*, v. 34, 1947, p. 139-167.

3. P. B. PATNAIK, "The power function of the test between two proportions in a 2×2 table," *Biometrika*, v. 35, 1948, p. 157-175.