

0, 1, 2 (whereas vol. 1 has $m = 0(1)5$). Otherwise, the range and intervals of this volume match those of the first.

Since $P_n^m(\cos \theta)$ has a singularity at $\theta = 180^\circ$ (except when n is an integer), an auxiliary function $T_n^m(\cos \theta)$ is introduced by the relation

$$P_n^m(\cos \theta) = (\csc^m \theta) T_n^m(\cos \theta) + (-1)^m (A_n \log_{10} \left(\cot \frac{\theta}{2} \right)) P_n^m(\cos(180^\circ - \theta))$$

where $A_n = (2 \sin n\pi)/(\pi \log_{10} e)$. The function $T_n^m(\cos \theta)$ is tabulated for $135^\circ \leq \theta \leq 180^\circ$. The use of these auxiliary functions is facilitated by the provision of tables of A_n , $\csc \theta$, $\csc^2 \theta$, and $\log_{10} \cot(\theta/2)$.

The introductory material contains formulas, an account of the tables, hints for interpolation, and level curves of $P_n(\cos \theta)$, $P_n^1(\cos \theta)$, and $P_n^2(\cos \theta)$.

A. ERDÉLYI

California Institute of Technology
Pasadena, California

64[L].—LOUIS ROBIN, *Fonctions sphérique de Legendre et fonctions sphéroïdales*, tome 3, Gauthier-Villars, Paris, 1959, viii + 289 p., 24 cm. Price 5500 F.

The first two volumes of this work were reviewed in *MTAC*, v. 13, p. 325f. The present, final, volume contains chapters VII to X.

Chapter VII is devoted to the addition theorems of Legendre functions. Both Legendre functions of the first and second kind are included, and two cases are distinguished according as the composite argument lies in the complex plane cut from $-\infty$ to $+1$ or else on the cut between -1 and 1 . Addition theorems are also developed for the associated Legendre functions of the first kind.

Chapter VIII is devoted to zeros of Legendre functions. First the zeros of $P_n^m(\mu)$ as functions of μ , for fixed real m and n are discussed, then the zeros of $P_{-1/2+i_p}^m(\mu)$ when m is an integer and p a fixed real number, and then the zeros of $Q_n^m(\mu)$. This chapter contains also a discussion of zeros of Legendre functions considered as functions of n , m and μ being fixed. (These zeros are of importance in certain boundary-value problems.)

In Chapter IX, applications of Legendre functions are given to partial differential equation problems relating to surfaces of revolution other than spheres. Prolate and oblate spheroidal harmonics, toroidal harmonics, and conal harmonics are discussed.

Chapter X contains the discussion of some functions related to Bessel functions, namely, Gegenbauer polynomials and functions, and spheroidal wave functions.

Appendix I summarizes relevant information on "spherical Bessel functions", and Appendix II lists numerical tables of Legendre functions and tables connected with these functions.

The third volume maintains the high standards set by the first two volumes, and the author must be congratulated upon the completion of this valuable work.

A. ERDÉLYI

California Institute Technology
Pasadena, California