65[L. M.].—R. G. SELFRIDGE & J. E. MAXFIELD, *A Table of the Incomplete Elliptic Integral of the Third Kind*, Dover Publications, Inc., 1959. xiv + 805 p., 22 cm. Price $7.50.

The advent of fast automatic computers has made a considerable difference to the art of making and publishing tables. It has speeded up the computing processes fantastically, without, except in relatively minor ways, modifying the labor and care needed during publication processes. It has also increased the care needed in *planning* calculations; the plan has to be quite precise and exact in all details for the machine to produce proper results, whereas in desk computation, the plan can be built up and modified as the work proceeds.

The glamour of fast computation has led quite a number of people to enter the table-making field; people who appear to imagine that the whole problem is simplified by automatic computers, who perhaps do not even realize the need to seek expert advice. It is with some reluctance, but with the feeling that it is an urgent duty that needs to be performed on general grounds, that I suggest that the table now reviewed presents one of the most deplorable examples of inadequate planning and poor execution that I have met.

The tables give entries that purport to be 6-decimal values of

$$\Pi(\phi, \alpha^2, k) = \int_0^\phi \frac{d\theta}{(1 - \alpha^2 \sin^2\theta)\sqrt{1 - k^2 \sin^2\theta}}$$

for

$$k = \sin \theta, \qquad \theta = .1(.1)1.5, \qquad \phi = 0(.01)1.57,$$

$$\alpha^2 = -1(.05) - .1(.02) - .02, .05(.05).5(.02).8(.01).99.$$

Also given are two lines for $\phi = 1.5707963$, representing a direct and a check calculation.

One example of bad planning is illustrated by the arguments θ, and the heading, erroneously labelled α. These end arbitrarily in 4 or 5 zeros or 4 or 5 nines. The latter is obviously wrong and quite intolerable in print. The present authors are not unique in exhibiting this fault, which is quite inexcusable. It is a matter of, at most, a few minutes to modify a program, on any automatic machine, to round-off at the appropriate figure and to suppress printing thereafter, giving better-looking and more convincing argument values. The authors have, in any case given special treatment to the argument θ which has one *more* decimal than function values; why not treat it properly?

Both bad planning and poor execution are exhibited by the check up. The foreword states that "the greatest difficulty was encountered not in constructing the table, but in obtaining satisfactory checking". This has not, in fact, been achieved! The method described could have been—but is not—satisfactory for $\alpha^2 < 0$ and for $0 < k^2 < \alpha^2$, but the method as described, of integrating through a singularity when $\alpha^2 > 0$, is absurdly inadequate. It is not surprising then, that the final two lines, both for $\phi = 1.5707963$ as mentioned before, should often be in disagreement. It is surprising, however, that the authors accept this as a legitimate problem to hand on to their readers. The discrepancies indicate errors; the authors' duty is to find and remove these. This states the obvious, but it is equally obviously necessary to do so. Let it be said again, that to make a program to deal with this properly is a job to be done once and for all. It may take a little effort, time, and money, but this is not to be compared with effort, time, and money wasted if it is *not* done before publishing a book of inadequate tables.

I believe, for my part, the electronic computation is so fast and easy that a discrepancy of more than a single unit (known or unknown) is not tolerable in a published table (of which the computing effort and cost are now often only a small part of the whole effort and cost); however, I am willing to concede that discrepancies up to perhaps two units *might*, on rare occasions, be justified, provided this is clearly stated. It is quite intolerable to have the following discrepancies; to pick out some of the worst:

Page	α^2	k^2	$\Pi(\alpha^2, k)$
661	0.87	0.86869...	8.654098 and 8.654259
733	0.93	0.92844...	15.309882 and 15.310251
745	0.94	0.92844...	16.857725 and 16.857859
793	0.98	0.97111...	45.498015 and 45.498457
805	0.99	0.92844...	49.243943 and 49.244046
805	0.99	0.97111...	70.018520 and 70.018897

The discrepancies are, in fact, highly systematic throughout; they are all of the same sign, except for the violent cases mostly listed above near the singularity $k = \alpha$ mentioned in the introduction. They indicate clearly that at least one set of the check values is erroneous because of an inadequate method, and not merely because of rounding; severe doubt is cast on *both* discrepant values. Dr. J. W. Wrench, Jr. has computed anew the value on p. 733 for $\alpha^2 = .93$, $k^2 = \sin^2 1.3$, and finds 15.3098662, which is not even between the two values quoted from the book; neither published value is correct and one errs by more than the discrepancy between them. I repeat again, it is the duty of the table compiler to remove *all* these doubts.

The poor execution is also exhibited by the fact that in the heading, α and k appear in place of the correct α^2 and k^2, while the 10-decimal values of k^2 given (which are simply $\sin^2 \lambda$ for $\lambda = .1(.1)1.5$) have end figure errors running up to 11 units. Again, the argument is given as θ in the tables; this corresponds to ϕ in the introduction. It is only fair to add that the heading errors in α^2, k^2, ϕ have been announced as errata. Another awkward point for the user is that absence of values for $\alpha = 0$, and for $k = 0$, makes the tables harder to interpolate.

From all this, it is evident that the authors are lacking in experience of table-making so that their remark "With the argument as outlined, no attempt has been made to proof or check the printed sheets in any way other than by comparison of the resultant complete integrals" causes less surprise than might otherwise be the case. If there were no other faults occurring other than those mentioned above, they would have been exceptionally and unduly lucky. Photographic processes seem as far from infallibility as printing from letter-press; the possible faults are different, but nevertheless exist just the same. In fact, a rather superficial examination of the table reveals unsightly irregularities in spacing of lines on pages 51, 623, 700 and some lesser ones elsewhere. It would have been easy to reprint the pages before reproduction. More serious are several digits that are not fully legible:

p. 446	$\theta = .25$	$k = .03946 \cdots$	3rd digit 3
p. 507	$\theta = .75$	$k = .15164 \cdots$	1st digit 8
p. 579	Bottom right corner.	Very faint.	

All these imperfections occur in at least two copies of the tables; such imperfections are common in tables printed from typescript and should be expected and sought out. The real surprise is, however, as mentioned above, that, after the numerical comparison of check values mentioned had been made, its lack of success seems simply to have been ignored.

It is hoped that possible users may, with the exercise of necessary—but undue—caution, obtain adequate results, maybe $5\frac{1}{2}$ correct figures, if they need them. The publication of this book will undoubtedly make it much more difficult to publish a good and proper version; this is a major criticism of such a book. The only consolation I can offer the authors is that I have seen several tables that are even worse.

As I have said, I have expressed myself so freely with some reluctance, from a sense of duty; it is no part of my desire to discourage the enthusiasm of table-makers, but they must realize the magnitude and duties of the task so taken on, and seek competent advice before proceeding with the work, and potential users must be adequately warned.

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66[S].—D. R. Hartree, *The Calculation of Atomic Structures*, John Wiley and Sons, Inc., New York, 1957, xiii + 181 p., 23 cm. Price $5.00.

This book by the late D. R. Hartree is the fruit of a lifetime of experience in the calculation of the outer, electronic structure of atoms. It is concerned with methods for the calculation of atomic structures rather than with the results of such calculations for particular atoms. Emphasis is deliberately placed on means of obtaining "best" approximations which can be both represented and applied simply. The student who wants an introduction to the essential methods of approximation and computation of shell structures may read the first hundred pages. The mathematician will find in this book the physical background for the author's well-known text on numerical analysis.

In the Introduction are outlined the seven main steps in the development of atomic theory up to the point at which quantitative calculations are possible. The atomic units are introduced and the point change approximation of the electron justified. Then, properties of the Schroedinger equation are summarized to prepare the reader for the main problem of the book, the numerical solution of the self-consistent field equations with and without exchange. The variation principle is carefully introduced, and the total energy of closed shell configurations discussed. Also, configurations comprising incomplete groups are treated. In the later part, the main ideas and methods are extended to more complicated or more complete