Computation of Fresnel Integrals

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Two approximations, one valid for x less than 4 and the other valid for x larger than 4, have been established by means of the τ-method of Lanczos [1] for the Fresnel integrals defined in the form

$$f(x) = \int_0^x \frac{e^{-it}}{\sqrt{2\pi t}}\,dt = C(x) - iS(x).$$

These approximations are the following:

(1) For $0 \leqq x \leqq 4$ $$f(x) = e^{-ix}\sqrt{\frac{x}{4}}\sum_{n=0}^{11}(a_n + ib_n)\left(\frac{x}{4}\right)^n$$

(2) For $x \geqq 4$ $$f(x) = \frac{1-i}{2} + e^{-ix}\sqrt{\frac{4}{x}}\sum_{n=0}^{11}(c_n + id_n)\left(\frac{4}{x}\right)^n.$$

The numerical values of the coefficients a_n, b_n, c_n and d_n are given by

$a_0 = +1.595769140$	$b_0 = -0.000000033$	$c_0 = 0$	$d_0 = +0.199471140$
$a_1 = -0.000001702$	$b_1 = +4.255387524$	$c_1 = -0.024933975$	$d_1 = +0.000000023$
$a_2 = -6.808568854$	$b_2 = -0.000092810$	$c_2 = +0.000003936$	$d_2 = -0.009351341$
$a_3 = -0.000576361$	$b_3 = -7.780020400$	$c_3 = +0.005770956$	$d_3 = +0.000023006$
$a_4 = +6.920691902$	$b_4 = -0.009520895$	$c_4 = +0.000689892$	$d_4 = +0.004851466$
$a_5 = -0.016898657$	$b_5 = +5.075161298$	$c_5 = -0.009497136$	$d_5 = +0.001903218$
$a_6 = -3.050485660$	$b_6 = -0.138341947$	$c_6 = +0.011948809$	$d_6 = -0.017122914$
$a_7 = -0.075752419$	$b_7 = -1.363729124$	$c_7 = -0.006748873$	$d_7 = +0.029064067$
$a_8 = +0.850663781$	$b_8 = -0.403349276$	$c_8 = +0.000246420$	$d_8 = -0.027928955$
$a_9 = -0.025639041$	$b_9 = +0.702222016$	$c_9 = +0.002102967$	$d_9 = +0.016497308$
$a_{10} = -0.150230960$	$b_{10} = -0.216195929$	$c_{10} = -0.001217930$	$d_{10} = -0.005598515$
$a_{11} = +0.034404779$	$b_{11} = +0.019547031$	$c_{11} = +0.000233939$	$d_{11} = +0.000838386$

The derivation of these approximations is given in [2].

The maximum error is 1.6×10^{-9} for the first approximation and 0.5×10^{-9} for the second approximation.

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1. C. Lanczos, *Applied Analysis*, Prentice Hall, Englewood Cliffs, N. J., 1956.
2. J. Boersma, "On a numerical method for the computation of Fresnel integrals", *Report TW 2*, Math. Inst., Univ. of Groningen, 1960.

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