94 [Q, S].—GARRETT BIRKHOFF AND R. E. LANGER, Editors, *Proceedings of Symposia in Applied Mathematics*, Vol. IX, "Orbit Theory," (Proceedings of the Ninth Symposium In Applied Mathematics of the American Mathematical Society, held at New York University April 4–6, 1957, cosponsored by The Office of Ordnance Research, Ordnance Corps, U. S. Army) American Mathematical Society, Providence, R. I., 1959, v + 195 p., 26 cm. Price $7.20.

The purpose of the book is, paraphrasing the words of the editors, to direct the attention of mathematicians to recent advances in celestial mechanics and, more importantly, to inform them of the problems that remain to be solved. Celestial mechanics owes its present form very largely to analysis, as it was developed in the eighteenth and nineteenth centuries. Whether modern mathematics can contribute anything important to the subject is a question that has hardly been explored, and it is high time that it should be.

Of the ten contributions by as many authors the first three deal with the motions of particles in magnetic fields, the remaining seven with motions of particles in gravitational fields. The magnetic fields considered are those in particle accelerators, in the galaxy, and about a laboratory model of the earth. The gravitational fields are principally those of the earth and of the solar system, although one paper deals generally with the field about any massive particle, and one with a general planetary system.

The various contributions are very uneven, ranging from rather trivial special applications of general formulae, through adaptations and modifications that are not trivial, to some important original contributions, both general and particular. Some authors describe what they have done themselves, some what others have done, and some what has not been done. Brouwer, Courant, and Olbert give special attention to unsolved problems; the references will be valuable to a mathematician not previously acquainted with their subjects. Herget and Eckert deal with practical computation.

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95[X].—RUDOLPH E. LANGER, Editor, *Boundary Problems in Differential Equations*, Proceedings of a symposium conducted by the Mathematics Research Center, University of Wisconsin, Madison, Wisconsin, The University of Wisconsin Press, Madison, 1960, x + 324 p., 24 cm. Price $4.00.

This volume contains the nineteen papers presented at the symposium on 'Boundary Problems in Differential Equations" conducted by the Mathematics Research Center at Madison, Wisconsin during the period April 20–22, 1959. The papers are quite varied in nature and subject matter, as is clear from the table of contents given below:

Complete Systems of Solutions for a Class of Singular Elliptic Partial Differential Equations
Peter Henrici, University of California
Application of the Theory of Monotonic Operators to Boundary Value Problems
Lothar Collatz, University of Hamburg, Germany
Upper and Lower Bounds for Quadratic Integrals and, at a Point, for Solutions of Linear Boundary Value Problems
J. B. Diaz, Institute for Fluid Dynamics and Applied Mathematics, University of Maryland
Error Estimates for Boundary Value Problems Using Fixed-Point Theorems
Johann Schroder, University of Hamburg, Germany
On a Unified Theory of Boundary Value Problems for Elliptic-Parabolic Equations of Second Order
Gaetano Fichera, The Mathematical Institute, University of Rome, Italy
Factorization and Normalized Iterative Methods
Richard S. Varga, Westinghouse Electric Corporation, Bettis Atomic Power Division, Pittsburgh
Some Numerical Studies of Iterative Methods for Solving Elliptic Difference Equations
David Young and Louis Ehrlich, The University of Texas.
Presented by David Young
Albedo Functions for Elliptic Equations
Garrett Birkhoff, Harvard University
A Numerical Method for Analytic Continuation
Jim Douglas, Jr., The Rice Institute, Texas
Stress Distribution in an Infinite Elastic Sheet with a Doubly-Periodic Set of Equal Holes
W. T. Koiter, Technical University, Delft, Holland
Some Stress Singularities and Their Computation by Means of Integral Equations
Hans F. Bueckner, Mathematics Research Center, U. S. Army
Boundary Value Problems in Thermoelasticity
Ian N. Sneddon, The Univerity, Glasgow, Scotland
Some Numerical Experiments with Eigenvalue Problems in Ordinary Differentia Equations
Leslie Fox, University Computing Laboratory, Oxford, England
Dynamic Programming, Invariant Imbedding, and Two-Point Boundary Value Problems
Richard Bellman, The Rand Corporation, California
Remarks about the Rayleigh-Ritz Method
Richard Courant, Institute of Mathematical Sciences, New York University
Free Oscillations of a Fluid in a Container
B. Andreas Troesch, Space Technology Laboratories, Inc., California
A Variational Method for Computing the Echo Area of a Lamina
Calvin H. Wilcox, Mathematics Research Center, U. S. Army, and California Institute of Technology

Workers in Numerical Analysis will be particularly interested in the papers of Friedrichs, Garabedian, Collatz, Varga, Schroder, Young and Ehrlich, Douglas, and Fox. The first author has a very short paper in which he gives an interesting outline of a unified approach to the numerical treatment of linear partial differential equations irrespective of their type. The unified approach is said to also cover certain equations of mixed type. Unfortunately, the author did not have space to completely describe the conditions he must impose on the equations he treats.

Garabedian describes a method that has been used to calculate axially symmetric flows with free streamlines. In particular he discusses methods for calculating the contraction coefficient in the vena contracta. The method involves generalizing the differential equation governing the flow by introducing a parameter λ and studying the dependence of the solution as a function of λ.

Collatz's paper is an expository one in which he discusses various definitions of monotonic operators and applies such definitions to the determination of bounds on the solutions of various problems.

Schroder uses monotonic operators which satisfy a fixed point theorem to prove the existence of solutions to problems involving differential equations and boundary conditions. He also determines approximate solutions and error bounds by solving a so-called comparison problem.

Varga discusses a class of iterative methods for solving a system of linear equations which depend on the direct solution of matrix equations of matrices more general than tridiagonal matrices. He shows how such matrix equations can be directly and efficiently solved and, in addition, applies standard methods for accelerating convergence.

Young and Ehrlich report on numerical experiments which attempted to determine the extent to which theoretical results on the rate of convergence of the successive over-relaxation method for solving linear equations and for the Peaceman-Rachford method would apply for non-rectangular regions. The theoretical results are known for the latter method only in the rectangular case. In nearly every case it was found that the number of iterations using the Peaceman-Rachford method was less than was required using the successive over-relaxation method. However, approximately three times as much computer time is required for a double sweep of the former method as is required for a single step of the latter method.

Douglas discusses the determination of an approximation of an analytic function of a complex variable inside the disk $0 \leqq |z| \leqq 1$ when bounds on the function and its first two derivatives are known and when approximate values of the functions are known at p points equidistributed on the circle $|z| = 1$. An estimate of the error of the approximation is also obtained.

Fox discusses a method for the determination of approximate proper values and proper solutions to single or systems of ordinary differential equations for which a reasonable approximation is already known for the proper value. The method involves the introduction of parameters such as initial values of the solution at one of the boundary points and the determination of improved values for these parameters by the Newton process. The method described is not new, but the applications made by Fox to fairly difficult problems give an impressive demonstration of its power.

Space limitations prevent the reviewing of the remaining papers in this volume.

They are of high quality. The organizers of the conference are to be congratulated on the papers solicited. The University of Wisconsin Press has produced a handsome volume by a photographic process which makes a very readable page. The relatively low cost of the volume is especially noteworthy.

A. H. T.

96[X].—W. L. Wilson, Jr., "Operators for solution of discrete Dirichlet and Plateau problems over a regular triangular grid," May 1959, 29 cm., 191 p. Deposited in UMT File.

These tables list to 10D coefficients of a matrix operator for conversion of boundary values over an equilateral triangle to a discrete harmonic function over a regular triangular grid of 190 points in this triangle [1]. Sixty-three boundary values are involved, of which the three at the vertices do not influence the interior values of the function. The tables are useful in the approximate numerical solution of the Laplace equation over this triangular region.

Solutions for smaller triangles have been placed in the UMT File by the same author [2].

Also included are tables giving 10D coefficients of the analog of the Douglas functional over this same grid. Specifically, these are coefficients of a quadratic form (using scalar multiplication) of vector functions from the grid points of the bounding equilateral triangle to some euclidean space such that the value of the form is the Dirichlet integral

$$D = \frac{1}{2} \int (E + G) \, d\sigma$$

where E and G are coefficients of the first fundamental form of the surface got by linear interpolation of the discrete harmonic vectors resulting from application of the operator described above to the boundary values. This is a discrete analog of the functional used by J. Douglas [3] in his solution of the Problem of Plateau; it has application in the approximate numerical solution of that problem.

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1. L. V. Kantorovich & K. I. Krylov (translated by Curtis D. Benster) *Approximate Methods of Higher Analysis*, Nordhoff, Gronigen, Interscience, New York, 1958, p. 187–188.
2. W. L. Wilson, Jr., "Tables of inverses to Laplacian operators over triangular grids," UMT File, MTAC, No. 58, v. XI, 1957, p. 108.
3. J. Douglas, "Solution of the problem of Plateau," Amer. Math. Soc. *Trans.*, v. 33, 1931, p. 263–321.

97[Z].—Jack Bonnell Dennis, *Mathematical Programming and Electrical Networks*, John Wiley & Sons, Inc., New York, 1959, vi + 186 p., 24 cm. Price $4.50.

As the title indicates, the purpose of this little monograph is to explore the relationships of general programming problems and corresponding electrical networks, with a view towards gaining physical insight and developing computational algorithms. The contents of the book essentially comprise the author's doctoral