

to the individual dice and to the group of three detected no bias. Therefore, it seems reasonable to assume that the variations from a true regular icosahedron are minor in terms of the application intended.

Less meticulous samplings have been made with the aged dice, but it seems unlikely that a person generating random decimal digits at a rate which can be met by these dice would notice any serious bias.

The frequencies with which the digits appear can be changed slightly by the usual standard means. These would include weighting to displace the center of gravity (an awkward and cumbersome method at best, and, at worst, one which is difficult to disguise) or applying wax to one or more faces to increase the probability that the waxed face will be on the bottom after the throw.

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1. THE RAND CORP., *One Million Random Digits and 100,000 Normal Deviates*, The Free Press, Glencoe, Illinois, 1955.

2. C. B. TOMPKINS, RMT 11, *MTAC*, v. 10, 1956, p. 39-43.

18[L].—F. M. HENDERSON, *Elliptic Functions with Complex Arguments*, The University of Michigan Press, Ann Arbor, 1960, v + 38 p. + 160 unnumbered p., 29 cm. Price \$8.00.

The tables and charts on the 160 unnumbered pages are in four equal parts, relating in turn to the functions $sn w$, $cn w$, $dn w$, and $E(w) = \int_0^w dn^2 w dw$, each for 19 values of the modular angle $\sin^{-1} k$, namely, 1° , 5° , 85° , 89° . Each opening has a table on the left and a corresponding chart on the right. If $w = u + iv$, the tables are all for $u/K = 0(.1)1$, $v/K' = 0(.1)1$, and the charts cover the same unit square on a scale such that the side of the square is about 16.2 cm. In Parts I-III, the quantities tabulated, to 4 figures without differences, are the real and imaginary parts x , y of $sn w$, $cn w$, $dn w$, respectively. In Part IV, if $E_R + iE_I = E(w)$, the relations $E(K) = E$, $E(K + iK') = E + i(K' - E')$ have led to the tabulation of the normalized quantities $E_R' = E_R/E$, $E_I' = E_I/(K' - E')$. The lines drawn on the charts are curves of constant x or y in Parts I-III, and curves of constant E_R' or E_I' in Part IV. Seven-figure values of the complete integrals are provided. The information given is sufficient to enable the four functions concerned to be evaluated for any point w in the complex plane.

The well-known tables of Spenceley and Spenceley [1] were used as a source of values for real w , whence the imaginary transformation and the addition formulas were used to compute the values of the functions of $u + iv$. The computation was mostly done on an IBM 650.

The Introduction contains enough information about elliptic integrals and functions to explain the tables and charts to anyone not previously acquainted with the subject. It also contains several applications to potential problems. It is pleasant to find such a valuable contribution to mathematical tabulation made by a civil engineer. The charts were constructed by a group of five Turkish naval officers at the University of Michigan.

One could wish that italic type (available and used in other contexts) had been used for mathematical symbols in the Introduction, but no such minor matter

can obscure the importance of the volume, which cannot fail to be found very useful. Only in the case of Part IV has the reviewer heard of any similar or related table of comparable scope, namely the table of the Jacobian zeta function by Fox and McNamee [2].

A. F.

1. G. W. SPENCELEY & R. M. SPENCELEY, *Smithsonian Elliptic Functions Tables*, Washington, 1947. See *MTAC*, v. 3, 1948, p. 89-92.

2. E. N. FOX & J. MCNAMEE, "The two-dimensional problem of seepage into a cofferdam," *Phil. Mag.*, s. 7, v. 39, 1948, p. 165-203. See *MTAC*, v. 3, 1948, p. 246, 252, also v. 7, 1953, p. 190.

19[L].—L. LEWIN, *Dilogarithms and Associated Functions*, MacDonald & Co. Ltd., London, 1958, xvi + 353 p., 21 cm. Price 65 Shillings.

The functions treated here are for the most part special cases of the Lerch zeta function, which can be defined by the series $\sum_{n=0}^{\infty} z^n / (n + b)^s$, $|z| < 1$, b not a negative integer or zero. To describe the text and tables, it is convenient to give some notation. Let $z = x + iy$, where x and y are real and $i = \sqrt{-1}$. Then

$$\begin{aligned}
 (1) \quad Li_2(z) &= - \int_0^z t^{-1} \ln(1-t) dt, & Li_n(z) &= \int_0^z t^{-1} Li_{n-1}(t) dt; \\
 (2) \quad Ti_2(x, a) &= \int_0^x (t+a)^{-1} \arctan t dt, & Ti_2(x) &\equiv Ti_2(x, 0), \\
 & & Li_n(iy) &= 2^{-n} Li_n(-y^2) + iTi_n(y); \\
 (3) \quad Cl_2(\theta) &= - \int_0^\theta \ln\left(2 \sin \frac{1}{2} t\right) dt, & Cl_{2n}(\theta) &= \int_0^\theta Cl_{2n-1}(t) dt, \\
 & & Cl_{2n+1}(\theta) &= Li_{2n+1}(1) - \int_0^\theta Cl_{2n}(z) dt; \\
 (4) \quad Gl_{2n}(\theta) + iCl_{2n}(\theta) &= \sum_{k=1}^{\infty} \frac{e^{ik\theta}}{k^{2n}}, & Cl_{2n+1}(\theta) + iGl_{2n+1}(\theta) &= \sum_{k=1}^{\infty} \frac{e^{ik\theta}}{k^{2n+1}}.
 \end{aligned}$$

The function $Gl_n(\theta)$ is a polynomial in θ of degree n .

Chapter I deals with the dilogarithm function $Li_2(x)$. The function $Ti_2(x)$ is considered in Chapter II; $Ti_2(x, a)$ in Chapter III. $Cl_2(\theta)$, θ real and positive, is Clausen's integral, and is studied in Chapter IV. Chapters V and VI take up $Li_n(z)$ for $n = 2$ and 3, respectively, and the analysis of this function for general values of n is the subject of Chapter VII. The general relations in (3) and (4) are also studied in this chapter. Chapter VIII deals with series expansions and integrals which can be expressed in terms of the basic functions in (1)-(4). Chapter IX is very useful. It is a compendium of results derived in the previous chapters. It also contains a survey of mathematical tables.

A description of the tabular material in this volume follows.

Table I.	$Li_n(x)$,	$n = 2(1)5$	
		$x = 0(.01)1.0$,	5D
Table II.	$Ti_n(y)$,	$n = 2(1)5$	
		$y = 0(.01)1.0$,	5D