

given several courses on the subject for economists, but the emphasis in this book is to bring out the mathematical structure of the theory. He has succeeded in this in an admirable way. The comparatively few applications are meant as illustrations rather than as developments of the empirical fields for which the theory of games is particularly suited. There is an extensive discussion of zero-sum two-person games, and of equilibrium points for non-cooperative games. The presentation of linear programming is also clear and rather exhaustive. The theory of cooperative games, that is, those in which the formation of coalitions is advantageous and allowable, and in which side-payments by the players are freely admitted, is developed in fair detail, even including a discussion of Shapley's value of an n -person game. At this point the author confesses that he is less sure of the intuitive background against which this theory has been placed, a position that is common to many mathematicians who have studied the problem of n -person games. However, this is a difficult issue and the author is wise not to have taken too definite a stand, rather withdrawing to the strictly mathematical aspects involved. These problems can only be solved by recourse to an improved description of the socio-economic world. If it turns out that the real problem involves a high degree of cooperation—and I have no doubts whatsoever that this will be the case—then the mathematical theory will have to accommodate itself to these facts, even if the mathematical structure is uncommon and cumbersome, until fundamentally new concepts are established.

Dr. Burger possesses a very high didactic skill; the great clarity which pervades his whole book should make it a welcome tool for the novice in game theory who commands the mathematical knowledge expected of first or second year graduate students. A translation of the work should be seriously considered, since there is no similar book in English which accomplishes as much in such small compass as Dr. Burger's does.

OSKAR MORGENSTERN

Princeton University
Princeton, N. J.

26[X].—R. P. BOAS, JR. & R. C. BUCK, *Polynomial Expansions of Analytic Functions*, Springer-Verlag, Berlin, 1958, viii + 77 p., 23 cm. Price DM 19.80.

A great part of numerical analysis is concerned with polynomial approximation to analytic functions, and so this booklet appears of immediate interest to the numerical analyst. However, with numerical analysis in its present state, it is more relevant for studies in the general theory of functions of a complex variable or in the theory of special functions.

Given a set of polynomials $\{p_n(z)\}$ and a function $f(z)$, it is reasonable to ask whether we can find coefficients $\{c_n\}$ such that

$$(1) \quad f(z) = \sum c_n p_n(z)$$

in some sense. This is the "expansion problem".

It is also reasonable to ask whether, given linear functionals $\{L_n(f)\}$, for example,

$$(2) \quad L_n(f) = f^{(n)}(0)$$

or

$$(3) \quad L_{2n}(f) = f^{(2n)}(0), \quad L_{2n+1}(f) = f^{(2n)}(1),$$

we can obtain a representation of f in the form (1). This is the "interpolation problem". The solution in case (2) is given by the Taylor expansion and that in (3) by the two-point Lidstone expansion

$$f(z) = \sum A_n(1-z)f^{(2n)}(0) + \sum A_n(z)f^{(2n)}(1),$$

where the polynomials $A_n(z)$ are defined by

$$\frac{\sinh zw}{\sinh w} = \sum A_n(z)w^{2n}.$$

This is valid, for instance, when $f(z)$ is an entire function of exponential type less than π .

These two problems are studied by a general method—"kernel expansion"—for wide classes of polynomials (defined by generating functions) and for the cases when $f(z)$ is entire, or regular at the origin.

The material is accessible to those familiar with the classical methods of complex variable theory, and its study by numerical analysts is recommended. It will, for instance, encourage us to get off the real axis, reveal some thought-provoking "bad examples" (e.g., non-uniqueness in (1)), and show us how mathematics should be written.

JOHN TODD

California Institute of Technology
Pasadena, California

27[X].—RUDOLPH E. LANGER, Editor, *Symposium on Numerical Approximation*, The University of Wisconsin Press, Madison, 1959, x + 462 p., 24 cm. Price \$4.50.

The present volume auspiciously launches the publication efforts of Professor Langer's group at the Mathematics Research Center. It contains the proceedings of a symposium conducted at the University of Wisconsin, April 21–23, 1958. The papers (original and expository) present such a well-rounded survey of numerical approximation because of the careful selection of the invited contributors. A listing of the authors and the titles of their papers will serve to indicate the level and scope of the work.

A. M. Ostrowski	On Trends and Problems in Numerical Approximation
R. C. Buck	Linear Spaces and Approximation Theory
Z. Kopal	Operational Methods in Numerical Analysis Based on Rational Approximations
P. J. Davis	On the Numerical Integration of Periodic Analytic Functions
H. E. Salzer	Some New Divided Difference Algorithms for Two Variables
P. C. Hammer	Numerical Evaluation of Multiple Integrals
M. Golomb	Optimal Approximation and Error Bounds
A. Sard	The Rationale of Approximation
J. L. Walsh	On Extremal Approximations
E. L. Stiefel	Numerical Methods of Tchebycheff Approximation
L. Fox	Minimax Methods in Table Construction