such approximations to the specific volume and specific enthalpy as functions of pressure and temperature are included in the collection.

A. H. T.

53[P, X, Z].—WARD C. SANGREN, *Digital Computers and Nuclear Reactor Calculations*, John Wiley & Sons, New York, 1960, xi + 208 p., 24 cm. Price $8.50.

As the author states in his preface, the primary objective of this book is to present to nuclear engineers and scientists an introduction to high speed reactor calculations. Since the appearance of the basic reference, *The Elements of Nuclear Reactor Theory* by Glasstone and Edlund, Van Nostrand, 1952, the entire complexion of actual reactor design calculations has changed as a result of the growth in speed and size of computing machines, and reactor design calculations represent today a significant part of scientific computing time on modern computers.

The outline of the book by chapters is

Chapter 1. Introduction
Chapter 2. Digital Computers
Chapter 3. Programming
Chapter 4. Numerical Analysis
Chapter 5. A Code for Fission-Product Poisoning
Chapter 6. Diffusion and Age-Diffusion Calculations
Chapter 7. Transport Equation—Monte Carlo
Chapter 8. Additional Reactor Calculations

In Chapter 1, the author reviews the tremendous parallel growth of high speed computing machines and nuclear reactors, and their present interplay. In Chapter 2, an introduction and description of present day computers is given. In Chapter 3, programming for computers is introduced. After some preliminary remarks (no proofs) about interpolation, numerical integration, matrices, etc., items which can be found in many well-known texts on elementary numerical analysis, the author treats in Chapter 4 the more relevant problem of the numerical approximation of partial differential equations by difference equations, and their solution by means of iterative methods. Also, the treatment of interface conditions, which arise naturally in heterogeneous reactors, is given.

In Chapter 5, a simple code for fission-product poisoning is followed from the physical and mathematical definitions through to the construction of a program in the Bell (Wolontis) system.

In Chapter 6, the longest chapter, the author describes diffusion calculations, extending from steady-state criticality problems for reactors to the solution of two- and three-dimensional multigroup diffusion equations. In Chapter 7, the S_n method of Carlson is described, along with the use of Monte Carlo methods for solving problems such as those encountered in shielding calculations.

In his primary aim, the author does succeed. Nevertheless, the reviewer, being quite familiar with this area, was most critical with respect to the age of the references, as most of the technical papers referred to had appeared prior to 1957. As no serious attempt was made to fill the gap between these earlier developments and the developments which have taken place in the reactor field in the last few years, many statements in the book are either somewhat obsolete or misleading. For example, the numerical inversion of tridiagonal matrix equations on page 74 by an

algorithm is not stated to be simply Gauss elimination applied to the matrix problem, and in fact the author states that this "method" has not appeared in textbooks as yet. The iterative methods of Young-Frankel, and Peaceman-Rachford are each discussed twice, (p. 84 and p. 144) and not one of the four definitions is completely accurate. The book is, however, the only existing bridge between *The Elements of Nuclear Reactor Theory* and present computational technique in the reactor field.

R. S. V.

54[S, W].—RONALD A. HOWARD, *Dynamic Programming and Markov Processes*, Technology Press & Wiley, New York, viii + 136 p., 23 cm. Price $5.75.

Consider a physical system S represented at any time t by a state vector $x(t)$. The classical description of the unfolding of the system over time uses an equation of the form $x(t) = F(x(s), s \leqq t)$, where F is a prescribed operation upon the function $x(s)$ for $s \leqq t$. In certain simple cases, this reduces to the usual vector differential equation $dx/dt = g(x)$, $x(0) = c$.

For a variety of reasons, it is sometimes preferable to renounce a deterministic description and to introduce stochastic variables. If we take $x(t)$ to be a vector whose i-th component is now the probability that the system is in state i at time t, and allow only discrete values of time, we can in many cases describe the behavior of the system over time quite simply by means of the equation $x(t + 1) = Ax(t)$. Here $A = (a_{ij})$, $i, j = 1, 2, \cdots, N$, is a transition matrix whose element a_{ij} is the probability that a system in state j at time t will be found in state i at time $t + 1$. Processes of this type are called Markov processes and are fundamental in modern mathematical physics.

So far we have assumed that the observer plays no role in the process. Let us now assume that in some fashion or other the observer has the power to choose the transition matrix A at each stage of the process. We call a process of this type a *Markovian decision process*. It is a special, and quite important, type of dynamic programming process; cf. Chapter XI of R. Bellman, *Dynamic Programming*, Princeton University Press, 1957.

Let us suppose that at any stage of the process, we have a choice of one of a set of matrices, $A(q) = (a_{ij}(q))$. Associated with each choice of q and initial state i is an expected single-stage return $b_i(q)$. We wish to determine a sequence of choices which will maximize the expected return from n stages of the process. Denoting the maximum expected return from an n-stage process by $f_i(n)$, the principle of optimality yields the functional equation

$$f_i(n) = \max_q \left[b_i(q) + \sum_{j=1}^{N} a_{ij}(q) f_j(n - 1)\right].$$

In this form, the determination of optimal policies and the maximum returns is easily accomplished by means of digital computers; see, for example S. Dreyfus, *J. Oper. Soc. of Great Britain*, 1958. Problems leading to similar equations, resolved in similar fashion, arise in the study of equipment replacement and in continuous form in the "optimal inventory" problem; see Chapter Five of the book mentioned above and K. D. Arrow, S. Karlin, and H. Scarf, *Studies in the Mathematical Theory of Inventory and Production*, Stanford University Press, 1959.