Divisors of Mersenne Numbers 10,000 < p < 15,000

By Sidney Kravitz

Riesel has previously presented [1] a table of the smallest factors, q, of the Mersenne numbers, $M_p = 2^p - 1$, p, a prime, subject to the conditions that $p <$

Divisors of Mersenne Numbers
q Divides $M_p = 2^p - 1$; $K = (q-1)/2p$

p	K	p	K	p	K	p	K	p	K
10,007	12	10,883	1	12,037	3	12,923	1	13,841	3
10,067	25	10,891	9	12,041	75	12,941	4	13,883	1
10,091	1	10,949	7	12,049	11	12,959	1	13,903	12
10,111	5	10,973	3	12,073	15	12,979	60	13,963	12
10,133	7	10,979	4	12,107	4	13,001	4	13,997	3
10,139	60	10,987	9	12,119	1	13,003	8	13,999	5
10,141	8	10,993	11	12,157	132	13,049	4	14,033	3
10,151	96	11,059	21	12,203	1	12,063	20	14,057	12
10,163	1	11,071	113	12,241	288	13,109	7	14,071	69
10,181	24	11,083	336	12,263	1	13,127	108	14,081	15
10,211	4	11,119	5	12,329	207	13,163	28	14,083	5
10,247	9	11,171	1	12,347	108	13,177	3	14,149	11
10,271	1	11,173	12	12,379	5	13,229	15	14,159	1
10,289	12	11,197	12	12,391	8	13,241	4	14,197	288
10,301	220	11,243	57	12,401	51	13,249	96	14,303	1
10,321	3	11,273	27	12,409	396	13,291	5	14,387	13
10,331	1	11,299	12	12,421	3	13,313	40	14,411	13
10,333	60	11,311	65	12,437	4	13,339	5	14,461	20
10,357	3	11,317	108	12,451	200	13,367	4	14,537	3
10,429	267	11,399	60	12,479	4	13,397	19	14,551	5
10,433	7	11,471	1	12,527	33	13,411	20	14,591	25
10,457	3	11,497	23	12,539	4	13,417	12	14,593	3
10,513	12	11,519	1	12,547	129	13,451	1	14,627	88
10,589	96	11,579	1	12,553	32	13,463	1	14,633	40
10,607	13	11,617	80	12,577	368	13,487	4	14,639	57
10,613	16	11,621	36	12,583	8	13,537	80	14,657	3
10,631	25	11,699	1	12,611	49	13,567	377	14,669	4
10,657	104	11,701	3	12,641	31	13,577	3	14,699	1
10,663	8	11,719	132	12,647	340	13,619	1	14,713	20
10,687	332	11,783	1	12,671	1	13,627	32	14,767	5
10,691	1	11,813	3	12,703	5	13,669	11	14,779	9
10,733	3	11,827	113	12,739	45	13,679	4	14,783	1
10,753	8	11,831	1	12,757	3	13,681	44	14,831	1
10,799	1	11,833	3	12,781	48	13,697	3	14,843	16
10,837	83	11,939	1	12,791	1	13,751	9	14,879	1
10,853	3	11,959	5	12,823	20	13,759	144	14,891	16
10,859	25	11,969	12	12,899	1	13,763	1	14,939	1
10,861	68	11,981	171	12,911	21	13,799	16	14,957	4
10,867	12	12,011	1						

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10,000 and $q < 10{,}485{,}760$. The purpose of this note is to present an extension of this table for $10{,}000 < p < 15{,}000$ and $q < 10{,}485{,}760$.

The table presents the value of K for which $q = 2Kp + 1$ is the smallest divisor of M_p rather than presenting the divisor, q. This has been done because, first, the value of K indicates more about the character of the divisor than q does, and, second, to save space. All primes between 10,000 and 15,000 were examined. If any such prime is not listed in the table, it means that $2^p - 1$ has no prime factor $< 10 \cdot 2^{20}$.

Several criteria have been discovered concerning the divisors of the Mersenne numbers, M_p. The best known of these (for $K = 1$) is due to Euler [2]. It states that if $p = 4L + 3$ and $q = 8L + 7$ are both primes then q divides M_p. For $K = 3$, Pellet was the first to state [3] that $q = 6p + 1$ divides M_p if q can be expressed in the form $4(2a + 1)^2 + 27b^2$, and p and q are both prime. For $K = 4$, Reuschle stated and Western proved [4] that $q = 8p + 1$ divides M_p if q can be expressed in the form $d^2 + 64(2c + 1)^2$ and p and q are both prime.

These calculations were performed on an IBM 650 system at Picatinny Arsenal. The program used was as follows: all prime factors q of M_p ($p > 2$) are of the form $q = 2Kp + 1$, and of one of the two forms $8L \pm 1$. Thus, either $K = 0 \bmod 4$ or $p + K = 0 \bmod 4$. Each prime p was expressed in binary form, $p = \sum_{i=0} a_i 2^i$, $a_i = 0$ or 1. The residues $R_i = R_{i-1}^2 \bmod q$, $R_0 = 2$, were found. Finally the residue $\prod_{i=0} (R_i)^{a_i} \bmod q$ was evaluated. If this product is congruent to one (mod q) then q is a divisor of M_p.

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1. H. Riesel, "Mersenne Numbers," *MTAC*, v. 12, July 1958, p. 207.
2. G. H. Hardy & E. M. Wright, *An Introduction to the Theory of Numbers*, 3rd edition, 1956, Oxford University Press, p. 80.
3. L. E. Dickson, *History of the Theory of Numbers*, v. I, Chelsea Publishing Co., 1952, p. 25.
4. A. E. Western, "Some criteria for the residues of 8th and other powers, "*Proc. London Math. Soc.* (2) 9 (1911) p. 244–272.

On a Theorem of Mann on Latin Squares

By R. T. Ostrowski and K. D. Van Duren

Mann [1] proved the following theorem: *If a Latin square L of order* $4t + 2$ *has a* $(2t + 1) \times (2t + 1)$ *block with as many as* $(2t + 1)^2 - t$ *cells containing digits in a list of* $2t + 1$, *then there exists no Latin square orthogonal to L.* This theorem seemed until lately to give theoretical evidence for the truth of Euler's conjecture that no pair of orthogonal Latin squares exists of any order $4t + 2$. Now that Euler's conjecture has been shown to be false [2], [3], a more detailed investigation of Latin squares of orders $4t + 2$ seems worthwhile.

The chief goal of the work reported in this note was to find an example indicating that Mann's theorem is the best possible of its type for order 10—or conversely, to

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