The program was run for some sixty Latin squares of order 10 known to have orthogonal mates. Of these only one had a sum of 22—and this occurred for only one partitioning of rows and columns into fives.

Another routine was written for the same computer to generate random Latin squares of order 10. Digits 0 through 9 were produced by an appropriately modified random number generator; cells were filled in sequentially subject to the required conditions, and changes made when completion became impossible. One hundred Latin squares were produced and processed by the first program. The highest sums of counts of five digits in 5×5 blocks were 19 for 76 Latin squares, 20 for 23 squares, and 21 for one; no Latin square of the hundred had a sum over 21. Of the hundred Latin squares, no two produced the same list of counts in 15,876 blocks; this evidence supports the claim of randomness. Before this study it had seemed plausible that most Latin squares of order 10 have sums over 22, since considerable searching by computer in recent years suggests that pairs of orthogonal Latin squares of order 10 are rare. Almost certainly there are necessary conditions for Latin squares of order 10 to possess orthogonal mates, aside from falsity of the hypothesis of Mann's theorem.

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Complete Factorization of $2^{159} - 1$

By K. R. Isemonger

Algebraic factors of $2^{159} - 1$ are $2^3 - 1 = 7$ and $2^{53} - 1 = 6361 \cdot 69431 \cdot 20394401$, the latter factorization having been first published by F. Landry in 1869.

R. M. Merson of Farnborough, Hants, England has found 13960201 to be a prime factor of $(2^{106} + 2^{53} + 1)/7 \cdot 6679$. The resulting quotient is the integer

$$N = 124\ 302\ 077\ 031\ 586\ 210\ 969,$$

whose factorization was completed by me on 15 May 1960.

The procedure employed was to exhibit N as the difference of two squares, namely,

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$$N = a^2 - b^2,$$

where

$$a = (2^3 \cdot 5 \cdot 3^2 \cdot 53^2)x + 11\ 150\ 802\ 925.$$

This representation of a can be deduced from theory presented by Kraitchik [1], combined with the fact that both -1 and 5 are quadratic residues of N, as established by suitable representations of N by quadratic forms.

Corresponding to $x = 102908$, $a^2 - N$ is the square of $b = 114674787084$. Hence, N is the difference of the squares of $a = 115215488845$ and of the preceding value of b. Thus

$$N = 540\ 701\ 761 \cdot 229\ 890\ 275\ 929.$$

The primality of each of these factors was determined in a similar manner. The factorization of $2^{159} - 1$ is, therefore, now complete.

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Two Formulas Relating to Elliptic Integrals of the Third Kind

By J. Boersma

Using Legendre's notation, the normal elliptic integral of the third kind is defined by the equation

$$\prod(\phi, \alpha^2, k) = \int_0^\phi \frac{d\theta}{(1 - \alpha^2 \sin^2 \theta)\sqrt{1 - k^2 \sin^2 \theta}}.$$

For $k^2 < 1$, the following expansion holds uniformly over the closed interval $0 \leqq \theta \leqq \frac{\pi}{2}$:

$$\frac{1}{\sqrt{1 - k^2 \sin^2 \theta}} = \sum_{m=0}^{\infty} \binom{-\frac{1}{2}}{m} (-1)^m k^{2m} \sin^{2m} \theta,$$

where $\binom{-\frac{1}{2}}{m} = \frac{(-\frac{1}{2})(-\frac{3}{2})\cdots(-\frac{1}{2} - m + 1)}{m!}$ for $m > 0$, and $\binom{-\frac{1}{2}}{0} = 1$.

The factor $\frac{1}{1 - \alpha^2 \sin^2 \theta}$ in the integrand is bounded for $-\infty < \alpha^2 < \frac{1}{\sin^2 \phi}$ and $0 \leqq \theta \leqq \phi$; consequently, the expanded integrand may be integrated term by term. Such integration yields the series

$$\prod (\phi, \alpha^2, k) = \sum_{m=0}^{\infty} b_m k^{2m},$$

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