tion, but rather an ambivalent one, of the 3-dimensional rotation group. Thus, there exist tensors of the unitary group which are not found amongst the tensors of the rotation group, and these tensors, the so-called spin-tensors of the rotation group, have their physical significance; on the other hand, all tensors of the rotation group are found among the tensors of the unitary group.

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68 [K].—E. D. BARRACLOUGH & E. S. PAGE, "Tables for Wald tests for the mean of a normal distribution," *Biometrika*, v. 46, 1959, p. 169–177.

Let x be normally distributed with unknown mean θ and known variance σ^2. The Wald sequential test for $\theta = \theta_0$ against the alternative $\theta = \theta_1(\theta_1 > \theta_0)$ consists in taking observation x_{n+1} as long as $a < (\theta_1 - \theta_0)/\sum_{i=1}^{n} x_i/\sigma^2 + n(\theta_0^2 - \theta_1^2)/2\sigma^2 < b$; sampling stopping when this relation first fails, with $\theta = \theta_0$ accepted if the left-hand inequality fails and $\theta = \theta_1$ accepted if the right-hand inequality fails. Let $Z = -a\sigma/(\theta_1 - \theta_0)$, $h = (b - a)\sigma/(\theta_1 - \theta_0)$, $P_- =$ probability of accepting $\theta = \theta_0$ when true, $P_+ =$ probability of accepting $\theta = \theta_0$ when $\theta = \theta_1$, $N_- =$ average sample number when $\theta = \theta_0$, $N_+ =$ average sample number when $\theta = \theta_1$, and $\mu = (\theta_1 - \theta_0)/2$. Table 1 of the Appendix contains 2D values of h and Z for $P_+ = .05, .10(.10) .70$, $P_- = .95, .99, .995, .999$, and $\mu = .25(.25)1.00$. The values of a and b are determined by h and z. For Table 2 of the Appendix (the same combination of values for P_+, P_-, μ occur as for Table 1), 2D values are given for N_+ and N_-. Charts I–IV of the Appendix contain curves of P_+ and P_- as functions of h and Z for given μ. These charts are obtained directly from Table 1 and can be used to determine the operating characteristics of the test for given a, b, σ, θ_0, and θ_1. Also, a comparison is made between Wald's approximations (to the operating characteristics and the average sample numbers) and the true values, for ten combinations of values for h, Z, μ (Text-Table 1). The conclusion reached is that Wald's approximations are not acceptably accurate for many applications when $\mu \geqq .25$.

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69 [K].—A. T. BHARUCHA-REID, *Elements of the Theory of Markov Processes and their Applications*, McGraw-Hill Book Co., Inc., New York, 1960, xi + 468 p., 24 cm. Price $11.50.

In recent years the notions of probability have become increasingly important in the building of models of the world around us. This is true, for example, in certain of the physical sciences, social sciences, and in the simulation of military and other operations. Sometimes probabilistic notions appear directly as basic ingredients of the model, sometimes indirectly as the result of applying Monte Carlo methods to the solution of certain types of functional equations.

The mathematical abstraction of an empirical process whose development is governed by probabilistic laws is known as a stochastic process. A special class of these processes are Markov processes in which the development subsequent to a time t depends (probabilistically) only upon the state of the process at t and not upon its previous history. Their theory has been developed extensively in the past three decades and they have enjoyed increasingly wide application. Although the subject has been masterly treated from an advanced standpoint in Doob's classical *Stochastic Processes*, there remains a dearth of textbooks in English which are accessible to the beginning graduate student who has not yet mastered the subtleties of modern analysis, especially measure theory. Notable exceptions, for the case of a finite number of possible states, are Feller's highly regarded *Introduction to Probability Theory and its Applications* and the recent book on *Finite Markov Chains* by Kemeny and Snell.

The present work is intended as a graduate-level text and reference in applied probability theory. According to the author's preface, its purpose is twofold: "first, to present a nonmeasure-theoretic introduction to Markov processes, and second, to give a formal treatment of mathematical models based on this theory which have been employed in various fields." Further, he states that the prerequisites are "a knowledge of elementary probability theory (the first nine chapters of Feller, say), mathematical statistics (Mood level), and analysis (Rudin level). Some knowledge of matrices and differential equations is also required."

Part I develops the theory of Markov processes $x(t)$, with particular reference to branching stochastic processes. Chapter 1, which lays the foundations for its successors, is devoted to the case wherein x assumes values in a discrete denumerable space, and the parameter t is likewise discrete. In Chapter 2 the stochastic variable x again ranges over a discrete denumerable space, but the parameter t is continuous. In Chapter 3 both x and t are continuous. Each chapter is introduced by a summary paragraph. Thereafter the subject matter is developed carefully, systematically, and lucidly. Each chapter is followed by a list of problems and by a bibliography which is intended to be both complete and up-to-date. In order to make use of the latter, one would need a command not only of the standard languages of science—English, French, German, and Russian—but also of such less familiar languages as Polish and Hungarian.

In Part II it is shown how the theory can be appled to problems in a variety of fields. These fields, together with a partial listing of the topics covered, are:

Biology—Growth of populations, epidemics, theory of gene frequencies

Physics—Cascade processes, radioactive transformations, particle counters, theory of nuclear reactors

Astronomy and Astrophysics—Fluctuations of brightness in the Milky Way, spatial distribution of galaxies, radiative transfer

Chemistry—Reaction kinetics

Operations Research—Queueing theory and some of its applications.

Extensive bibliographies are also given for each of these topics.

The book closes with three appendices concerned with generating functions, Laplace and Mellin transforms, and Monte Carlo methods in the study of stochastic processes.

The reviewer was very favorably impressed by this book. The quality of the printing is excellent, with pleasing page layout and very few printing errors. The author's didactic skill and wide familiarity with the subject matter have resulted in a well motivated, well organized, clearly written text from which a student who has the indicated prerequisites can readily gain an introduction to the theory and applications of Markov processes.

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70 [K].—R. E. Beckhofer, Salah Elmaghraby & Norman Morse, "A single-sample multiple-decision procedure for selecting the multinomial event which has the highest probability," *Ann. Math. Statist.*, v. 30, 1959, p. 102–119.

Consider N k-nomial trials whose cell probabilities satisfy $p_1 = \cdots = p_{k-1} = p_k/\theta^*$. We select that cell into which the most events fall, breaking a tie at random if it occurs. The authors give a 5D table of the probability of selecting cell k, for $k = 2, 3, 4$; $\theta^* = 1.02(.02)1.1(.1)2(.2)3, 10$; and $N = 1(1)30$. An approximation is developed and compared with these values.

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71 [K].—K. G. Clemans, "Confidence limits in the case of the geometric distribution," *Biometrika*, v. 46, 1959, p. 260–264.

The author obtains confidence limits for estimating m, the expected number of trials before a device fails, given the sample mean $\bar{x}$, and N, the number of devices. If N devices each are from an identical geometric distribution, the distribution of sample sums will follow a Pascal distribution. Two log-log charts are provided for two-sided 90% and 98% confidence limits for m, $1 \leqq \bar{x} \leqq 10{,}000$, and $N = 2, 5, 10, 15, 20, 30, 50, 100$. The charts are based on the exact distribution. For $\bar{x} > 10{,}000$, formulas and tables may be used to determine the confidence limits. For large $N > 100$ a special formula is given. Alternatively for large N, since sample means are approximately normal, confidence limits for m may be found as solutions of the quadratic equation obtained from $t = \sqrt{N}(\bar{x} - m) \div m(m + 1)$, where t is the usual normal deviate for the α percent point.

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72 [K].—E. T. Federighi, "Extended tables of the percentage points of Student's t-distribution," *J. Amer. Statist. Assn.*, v. 54, 1959, p. 683–688.

The author states that in using Student's t-distribution in testing component parts a need for extending the table of upper percentage points was revealed. The method of calculation of these percentage points is presented, and a table containing these results is given. Let y_t be the elementary density for Student's t with n degrees of freedom, and denote $\int_{t_0}^{\infty} y_t\, dt$ by P. The values of t_0 are given to 3D for $P = .25, .10, .05, .025, .01, .005, .0025, .001, 5 \times 10^{-4}, 25 \times 10^{-5}, 1 \times 10^{-4}, 5 \times 10^{-5}, 25 \times 10^{-6}, 1 \times 10^{-5}, 5 \times 10^{-6}, 25 \times 10^{-7}, 1 \times 10^{-6}, 25 \times 10^{-8}, 1 \times 10^{-7}$, and $n = 1(1)\ 30\ (5)\ 60(10)\ 100, 200, 500, 10^3, 2 \times 10^3, 10^4$, and ∞. It would have been