

has the indicated prerequisites can readily gain an introduction to the theory and applications of Markov processes.

R. P. EDDY

Applied Mathematics Laboratory
David Taylor Model Basin
Washington 7, D. C.

70 [K].—R. E. BECKHOFER, SALAH ELMAGHRABY & NORMAN MORSE, “A single-sample multiple-decision procedure for selecting the multinomial event which has the highest probability,” *Ann. Math. Statist.*, v. 30, 1959, p. 102–119.

Consider N k -nomial trials whose cell probabilities satisfy $p_1 = \cdots = p_{k-1} = p_k/\theta^*$. We select that cell into which the most events fall, breaking a tie at random if it occurs. The authors give a 5D table of the probability of selecting cell k , for $k = 2, 3, 4$; $\theta^* = 1.02(.02)1.1(.1)2(.2)3, 10$; and $N = 1(1)30$. An approximation is developed and compared with these values.

J. L. HODGES, JR.

University of California
Berkeley, California

71 [K].—K. G. CLEMANS, “Confidence limits in the case of the geometric distribution,” *Biometrika*, v. 46, 1959, p. 260–264.

The author obtains confidence limits for estimating m , the expected number of trials before a device fails, given the sample mean \bar{x} , and N , the number of devices. If N devices each are from an identical geometric distribution, the distribution of sample sums will follow a Pascal distribution. Two log-log charts are provided for two-sided 90% and 98% confidence limits for m , $1 \leq \bar{x} \leq 10,000$, and $N = 2, 5, 10, 15, 20, 30, 50, 100$. The charts are based on the exact distribution. For $\bar{x} > 10,000$, formulas and tables may be used to determine the confidence limits. For large $N > 100$ a special formula is given. Alternatively for large N , since sample means are approximately normal, confidence limits for m may be found as solutions of the quadratic equation obtained from $t = \sqrt{N}(\bar{x} - m) \div m(m + 1)$, where t is the usual normal deviate for the α percent point.

L. A. AROIAN

Space Technology Laboratories, Inc.
Los Angeles, California

72 [K].—E. T. FEDERIGHI, “Extended tables of the percentage points of Student’s t -distribution,” *J. Amer. Statist. Assn.*, v. 54, 1959, p. 683–688.

The author states that in using Student’s t -distribution in testing component parts a need for extending the table of upper percentage points was revealed. The method of calculation of these percentage points is presented, and a table containing these results is given. Let y_t be the elementary density for Student’s t with n degrees of freedom, and denote $\int_{t_0}^{\infty} y_t dt$ by P . The values of t_0 are given to 3D for $P = .25, .10, .05, .025, .01, .005, .0025, .001, 5 \times 10^{-4}, 25 \times 10^{-5}, 1 \times 10^{-4}, 5 \times 10^{-5}, 25 \times 10^{-6}, 1 \times 10^{-5}, 5 \times 10^{-6}, 25 \times 10^{-7}, 1 \times 10^{-6}, 25 \times 10^{-8}, 1 \times 10^{-7}$, and $n = 1(1) 30 (5) 60(10) 100, 200, 500, 10^3, 2 \times 10^3, 10^4$, and ∞ . It would have been

advantageous had the large values of n been arranged conveniently for harmonic interpolation, such as $n = 60, 120, 240, 480, 960$, etc.

L. A. AROIAN

73[K].—IRWIN GUTTMAN, "Optimum tolerance regions and power when sampling from some non-normal universes," *Ann. Math. Statist.*, v. 30, 1959, p. 926–938.

This paper is concerned with obtaining β -expectation tolerance regions which are minimax and most stringent (see [1] and [2]) for the upper tail of the single exponential population and for the central part of the double exponential distribution. The single exponential probability density function (*pdf*) is of the form $\sigma^{-1} \exp [-(x - \mu)/\sigma]$ with $x \geq \mu$, where one or both of μ and σ are unknown. The double exponential *pdf* is of the form $(2\sigma)^{-1} \exp (-|x - \mu|/\sigma)$, where μ is known and σ is unknown. The sample values are $x_1 < \dots < x_n$; $\bar{x} = \sum_{i=1}^n x_i/n$; $s = \sum_{i=2}^n (x_i - x_1)/(n - 1)$; μ_0 and σ_0 represent known values of μ and σ ; $t = \sum_{i=1}^n |x_i - \mu_0|$. Then the optimum tolerance intervals, which are easily identified with the situations considered, are $[a_\beta(\bar{x} - \mu_0), \infty)$, $[x_1 - b_\beta\sigma_0, \infty)$, $[x_1 - c_\beta s, \infty)$, and $[\mu_0 - d_\beta t, \mu_0 + d_\beta t]$. Tables I–IV contain 6D values of a_β , b_β , c_β , d_β , respectively, for $n = 1(1)20, 40, 60$ and $\beta = .75, .90, .95, .99$. The power of tolerance intervals is expressed in terms of parameter α_1 , where α_1 is determined as the solution of $(\alpha\sigma)^{-1} \int_{I(\beta)} \exp [-(x - \mu)/\alpha\sigma] dx = \gamma = \text{measure of desirability}$,

for the single-exponential case, and from $(2\alpha\sigma)^{-1} \int_{I(\beta)} \exp (-|x - \mu|/\alpha\sigma) dx = \gamma$ for the double exponential case. Here $I(\beta)$ is the tolerance interval considered and $0 < \gamma < 1$ (large values indicate greatest desirability). Tables V, VI, and VIII contain 7D values of the power for intervals $[a_\beta(\bar{x} - \mu_0), \infty)$, $[x_1 - b_\beta\sigma_0, \infty)$, $[\mu_0 - d_\beta t, \mu_0 + d_\beta t]$, respectively, for $n = 1(2)7, 10, 15, 30, 60$, and $\beta = .75, .90, .95, .99$; likewise for $x_1 c_\beta s$ and Table VII, except that $n = 2(2)10, 15, 30, 60$.

J. E. WALSH

1. D. A. S. FRASER & IRWIN GUTTMAN, "Tolerance regions," *Ann. Math. Statist.*, v. 27, 1956, p. 162–179.

2. IRWIN GUTTMAN, "On the power of optimum tolerance regions when sampling from normal distributions," *Ann. Math. Statist.*, v. 28, 1957, p. 773–778.

74[K].—MILOS JILEK & OTAKAR LIKAR, "Coefficients for the determination of one-sided tolerance limits of normal distribution," *Ann. Inst. Statist. Math. Tokyo* v. 11, 1959, p. 45–48.

It is well known that a random sample of size N from a normal universe with mean μ and variance σ^2 yields one-sided tolerance limits $(-\infty, T_u)$ and $(T_L, +\infty)$ each of which includes at least a fraction α of the universe with probability P , where

$$T_u = \bar{x} + ks,$$

$$T_L = \bar{x} - ks,$$