advantageous had the large values of n been arranged conveniently for harmonic interpolation, such as $n = 60, 120, 240, 480, 960$, etc.

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73[K].—IRWIN GUTTMAN, "Optimum tolerance regions and power when sampling from some non-normal universes," *Ann. Math. Statist.*, v. 30, 1959, p. 926–938.

This paper is concerned with obtaining β-expectation tolerance regions which are minimax and most stringent (see [1] and [2]) for the upper tail of the single exponential population and for the central part of the double exponential distribution. The single exponential probability density function (*pdf*) is of the form $\sigma^{-1} \exp [-(x - \mu)/\sigma]$ with $x \geqq \mu$, where one or both of μ and σ are unknown. The double exponential *pdf* is of the form $(2\sigma)^{-1} \exp (- | x - \mu |/\sigma)$, where μ is known and σ is unknown. The sample values are $x_1 < \cdots < x_n$; $\bar{x} = \sum_{i=1}^n x_i/n$; $s = \sum_{i=2}^n (x_i - x_1)/(n - 1)$; μ_0 and σ_0 represent known values of μ and σ; $t = \sum_{i=1}^n | x_i - \mu_0 |$. Then the optimum tolerance intervals, which are easily identified with the situations considered, are $[a_\beta(\bar{x} - \mu_0), \infty)$, $[x_1 - b_\beta\sigma_0, \infty)$, $[x_1 - c_\beta s, \infty)$, and $[\mu_0 - d_\beta t, \mu_0 + d_\beta t]$. Tables I–IV contain 6D values of a_β, b_β, c_β, d_β, respectively, for $n = 1(1)20, 40, 60$ and $\beta = .75, .90, .95, .99$. The power of tolerance intervals is expressed in terms of parameter α_1, where α_1 is determined as the solution of $(\alpha\sigma)^{-1} \int_{I(\beta)} \exp [-(x - \mu)/\alpha\sigma\, dx = \gamma =$ measure of desirability, for the single exponential case, and from $(2\alpha\sigma)^{-1} \int_{I(\beta)} \exp (-| x - \mu | \alpha\sigma)\, dx = \gamma$ for the double exponential case. Here $I(\beta)$ is the tolerance interval considered and $0 < \gamma < 1$ (large values indicate greatest desirability). Tables V, VI, and VIII contain 7D values of the power for intervals $[a_\beta(\bar{x} - \mu_0), \infty)$, $[x_1 - b_\beta\sigma_0, \infty)$, $[\mu_0 - d_\beta t, \mu_0 + d_\beta t]$, respectively, for $n = 1(2)7, 10, 15, 30, 60$, and $\beta = .75, .90, .95, .99$; likewise for $x_1 c_\beta s$ and Table VII, except that $n = 2(2)10, 15, 30, 60$.

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1. D. A. S. FRASER & IRWIN GUTTMAN, "Tolerance regions," *Ann. Math. Statist.*, v. 27, 1956, p. 162–179.
2. IRWIN GUTTMAN, "On the power of optimum tolerance regions when sampling from normal distributions," *Ann. Math Statist.*, v. 28, 1957, p. 773–778.

74[K].—MILOS JILEK & OTAKAR LIKAR, "Coefficients for the determination of one-sided tolerance limits of normal distribution," *Ann. Inst. Statist. Math. Tokyo* v. 11, 1959, p. 45–48.

It is well known that a random sample of size N from a normal universe with mean μ and variance σ^2 yields one-sided tolerance limits $(-\infty, T_u)$ and $(T_L, +\infty)$ each of which includes at least a fraction α of the universe with probability P, where

$$T_u = \bar{x} + ks,$$

$$T_L = \bar{x} - ks,$$

and where,

$$\bar{x} = \sum_{i=1}^{n} x_i/n,$$

$$s^2 = \sum_{i=1}^{n} (x_i - \bar{x})^2/(n - 1),$$

and

$$\sqrt{n}k = t(n - 1, u_\alpha\sqrt{n}, 1 - P).$$

Here $t(f, \delta, \epsilon)$ [1] is the 100ϵ percentage point of the non-central t distribution with f degrees of freedom, δ is the measure of non-centrality in the definition of t, and u_α is the $100(1 - \alpha)$ percentage point of the unit normal distribution with zero mean.

By use of the tables (especially Table IV) and iteration of the approximations given by Johnson and Welch in [1] the authors obtain values of the coefficient $\sqrt{n}k$ to 4S, for $n = 5(1)20(5)50(10)\ 100(100)\ 300$, for P and $\alpha = .90, .95, .99$. A method for determination of these coefficients is given in [1], but the calculations are, of course, quite tedious, so that the present tables render a valuable service for practical applications to one-sided tolerance limits.

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75[K].—J. PACHARES, "Tables of the upper 10% points of the Studentized range," *Biometrika*, v. 46, 1959, p. 461–466.

Let $q = w/s$, where w is a sample range based on n values, and s is an independent estimate of standard deviation based on m values. Then tables of q' have been prepared for $Pr(q \geqq q') = \alpha$, where $\alpha = .01, .05, .10$, $n = 2\ (1)\ 20$, and $m = 1\ (1)\ 20, 24, 30, 40, 60, 120, \infty$. Three significant figures are given throughout. The work of Harter [1] has been used in improving the accuracy throughout, particularly for $\alpha = .01$. For $\alpha = .01$ and $.05$, these tables correct errors in [2].

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2. J. M. MAY, "Extended and corrected tables of the upper percentage points of the 'Studentized' range," *Biometrika*, v. 39, 1952, p. 192–193. [**RMT** 1080, *MTAC*, v. 7, 1953, p. 94]

76[K].—K. C. S. PILLAI & PABLO SAMSON, JR., "On Hotelling's generalization of T^2," *Biometrika*, v. 46, 1959, p. 160–168.

Let S_1/n_1, S_2/n_2 denote independent covariance matrices arising from samples of sizes n_1 and n_2 from two p-variate normal populations, and $U^{(s)} = \text{trace } S_2^{-1}S_1$, where s is the number of non-zero roots. Two approximations are compared with the