79[L].—L. Fox, *Tables of Weber Parabolic Cylinder Functions and Other Functions for Large Arguments*, National Physical Laboratory Mathematical Tables Volume 4, Her Majesty's Stationery Office, London, 1960, iii + 40 p., 28 cm. (Paperback) Price 12s. 6d.

In a previous work [1], to tabulate, for instance, $I_n(x)$ for large x, it was found convenient to write $I_n(x) = (2\pi x)^{-1/2}e^x F_n(x)$ and to tabulate the auxiliary function $F_n(x)$ for $1/x = z = 0(0.001)0.05$. This device is very economical as compared with tabulation as a function of x, and the $F_n(x)$ entries are easily interpolated. Using the same idea, the present volume gives tables which supplement well-known tables of certain transcendental functions as described below. An introduction describes the methods of computation. For each table second central or modified central differences are provided. In some cases modified fourth-order central differences are also given.

Table 1. The exponential integral

$$Ei(x) = \int_{-\infty}^{x} t^{-1}e^t\,dt, \qquad -Ei(-x) = \int_x^{\infty} t^{-1}e^{-t}\,dt$$

$$Ei(x) = e^x F(z), \qquad z = x^{-1}.$$

Thus, positive z relates to $Ei(x)$; negative z, to $Ei(-x)$. The function F is tabulated to 10D for $\pm z = 0(0.001)0.100$.

Table 2. Sine and cosine integrals

$$Si(x) = \int_0^x t^{-1}\sin t\,dt, \qquad Ci(x) = \int_\infty^x t^{-1}\cos t\,dt$$

$$Si(x) = \tfrac{1}{2}\pi - P\cos x - Q\sin x, \qquad Ci(x) = P\sin x - Q\cos x$$

Values of P, Q are given to 10D for $z = x^{-1} = 0(0.001)0.100$.

Table 3. Airy integrals

The notation follows Miller [2]. Let $z = \xi^{-1} = \tfrac{3}{2}x^{-3/2}$.

$$Ai(x) = \tfrac{1}{2}\pi^{-1/2}x^{-1/4}e^{-\xi}R, \qquad Bi(x) = \pi^{-1/2}x^{-1/4}e^{\xi}S$$

$$A'i(x) = \tfrac{1}{2}\pi^{-1/2}x^{1/4}e^{-\xi}W, \qquad B'i(x) = \pi^{-1/2}x^{1/4}e^{\xi}X$$

$$Ai(-x) + jBi(-x) = \pi^{-1/2}x^{-1/4}e^{-j\theta}(jP - Q)$$

$$A'i(-x) + jB'i(-x) = \pi^{-1/2}x^{1/4}e^{-j\theta}(jU - V), \qquad j = (-1)^{1/2},\ \theta = \xi + \tfrac{1}{4}\pi.$$

The values of R, S, W, X, P, Q, U, V are tabulated to 10D for $z = 0(0.001)0.050$.

Table 4. The error integral

$$\int_x^\infty e^{-\frac{1}{2}t^2}\,dt = x^{-1}e^{-\frac{1}{2}x^2}S, \qquad \int_0^x e^{\frac{1}{2}t^2}\,dt = x^{-1}e^{\frac{1}{2}x^2}T$$

The values of S and T to 10D are provided for $x^{-2} = z = 0(0.001)0.010$. Since error functions are often used in the form $\int^x e^{\pm t^2}\,dt$, tables based on the latter representation should also have been prepared.

Table 5. Factorial functions

This is a table of the gamma function, its natural logarithm, and derivatives of the latter. Let

$$\Gamma(1 + x) = (2\pi)^{1/2}e^{-x}x^{x+1/2}f,$$

$$\ln \{\Gamma(1 + x)\} = \tfrac{1}{2} \ln (2\pi) + (x + \tfrac{1}{2}) \ln x - x + g,$$

$$F^{(k-1)}(x) = \frac{d^k}{dx^k} \{\ln \Gamma(1 + x)\}, \qquad F^{(0)}(x) = \ln x + f_2, \qquad x^{-1} = z.$$

For each table $z = 0(0.01)0.10$. Tabular value of f, xf_2, xF', and x^2F'' are given to 10D; g, to 12D; x^3F''', $x^4F^{(4)}$, to 9D.

Table 6. Weber functions

The notation follows Miller [3]. Let

$$W(a, x) = (2k/x)^{1/2}f \cos \chi, \qquad W(a, -x) = (2/kx)^{1/2}f \sin \chi$$

$$\frac{d}{dx} W(a, x) = -(kx/2)^{1/2}g \cos \psi, \qquad \frac{d}{dx} W(a, -x) = -(x/2k)^{1/2}g \sin \psi,$$

$$\chi = \varphi + \tfrac{1}{4}x^2 - a \ln x + \tfrac{1}{2}\varphi_2 + \tfrac{1}{4}\pi, \qquad \psi = \omega + \tfrac{1}{4}x^2 - a \ln x + \tfrac{1}{2}\varphi_2 - \tfrac{1}{4}\pi$$

$$z = x^{-1}, \qquad k = (1 + e^{2\pi a})^{1/2} - e^{\pi a}, \qquad \varphi_2 = Im \ln \{\Gamma(\tfrac{1}{2} + ia)\}.$$

Values of f, φ, g, ω to 8D are tabulated for $a = -10(1)10$, $z = 0(0.005)0.100$. Values of k and φ_2 are also provided. Table 6A gives 8D values of φ_2 for $a = 0(0.05)2.50(0.1)10.0$.

Y.L.L.

1. L. Fox, *A Short Table for Bessel Functions of Integer Order and Large Arguments*, Royal Society Shorter Mathematical Tables, No. 3, Cambridge, 1954. See also Review 37, *MTAC*, v. 9, 1955, p. 73–74.

2. J. C. P. Miller, *The Airy Integral, giving Tables of Solutions of the Differential Equation* $y'' = xy$, British Association Mathematical Tables, Part-Volume B, Cambridge, 1946. See also Review 413, *MTAC*, v. 2, 1946–47, p. 302.

3. National Physical Laboratory, *Tables of Weber Parabolic Cylinder Functions*. Computed by Scientific Computing Service Limited; Mathematical Introduction by J. C. P. Miller, Editor. Her Majesty's Stationery Office, London, 1955. See also Review 101, *MTAC*, v. 10, 1956, p. 245–246.

80[X].—Germund Dahlquist, "Stability and error bounds in the numerical integration of ordinary differential equations," *Kungl. Tekn. Högsk. Handl. Stockholm* (Transactions of the Royal Institute of Technology, Stockholm, Sweden) Nr. 130, 1959, 85 p., 25 cm. Price Kr. 9.

In the first part of his thesis, the author investigates stability of certain linear operators,

$$L = \rho(E) - h^r (d^r/dx^r)\sigma(E) + h^{r+1} (d^{r+1}/dx^{r+1})\tau(E), \tag{1}$$

associated with numerical integration formulas for sets of differential equations of the form

$$d^r\bar{y}/dx^r = \bar{f}(x, \bar{y}), \tag{2}$$

where $\bar{y}$ and $\bar{f}$ are s-dimensional vectors, $\rho(\zeta)$, $\sigma(\zeta)$, and $\tau(\zeta)$ are polynomials of degree k with real coefficients, and E is the displacement operator defined by $Eu(x) = u(x + h)$, for any function $u(x)$. The number k is called the order of L.