the latter. Let

$$\Gamma(1 + x) = (2\pi)^{1/2}e^{-x}x^{x+1/2}f,$$

$$\ln \{\Gamma(1 + x)\} = \tfrac{1}{2} \ln (2\pi) + (x + \tfrac{1}{2}) \ln x - x + g,$$

$$F^{(k-1)}(x) = \frac{d^k}{dx^k} \{\ln \Gamma(1 + x)\}, \qquad F^{(0)}(x) = \ln x + f_2, \qquad x^{-1} = z.$$

For each table $z = 0(0.01)0.10$. Tabular value of f, xf_2, xF', and x^2F'' are given to 10D; g, to 12D; x^3F''', $x^4F^{(4)}$, to 9D.

Table 6. Weber functions

The notation follows Miller [3]. Let

$$W(a, x) = (2k/x)^{1/2}f \cos \chi, \qquad W(a, -x) = (2/kx)^{1/2}f \sin \chi$$

$$\frac{d}{dx} W(a, x) = -(kx/2)^{1/2}g \cos \psi, \qquad \frac{d}{dx} W(a, -x) = -(x/2k)^{1/2}g \sin \psi,$$

$$\chi = \varphi + \tfrac{1}{4}x^2 - a \ln x + \tfrac{1}{2}\varphi_2 + \tfrac{1}{4}\pi, \qquad \psi = \omega + \tfrac{1}{4}x^2 - a \ln x + \tfrac{1}{2}\varphi_2 - \tfrac{1}{4}\pi$$

$$z = x^{-1}, \qquad k = (1 + e^{2\pi a})^{1/2} - e^{\pi a}, \qquad \varphi_2 = Im \ln \{\Gamma(\tfrac{1}{2} + ia)\}.$$

Values of f, φ, g, ω to 8D are tabulated for $a = -10(1)10$, $z = 0(0.005)0.100$. Values of k and φ_2 are also provided. Table 6A gives 8D values of φ_2 for $a = 0(0.05)2.50(0.1)10.0$.

Y.L.L.

1. L. Fox, *A Short Table for Bessel Functions of Integer Order and Large Arguments*, Royal Society Shorter Mathematical Tables, No. 3, Cambridge, 1954. See also Review 37, *MTAC*, v. 9, 1955, p. 73–74.

2. J. C. P. Miller, *The Airy Integral, giving Tables of Solutions of the Differential Equation* $y'' = xy$, British Association Mathematical Tables, Part-Volume B, Cambridge, 1946. See also Review 413, *MTAC*, v. 2, 1946–47, p. 302.

3. National Physical Laboratory, *Tables of Weber Parabolic Cylinder Functions*. Computed by Scientific Computing Service Limited; Mathematical Introduction by J. C. P. Miller, Editor. Her Majesty's Stationery Office, London, 1955. See also Review 101, *MTAC*, v. 10, 1956, p. 245–246.

80[X].—Germund Dahlquist, "Stability and error bounds in the numerical integration of ordinary differential equations," *Kungl. Tekn. Högsk. Handl. Stockholm* (Transactions of the Royal Institute of Technology, Stockholm, Sweden) Nr. 130, 1959, 85 p., 25 cm. Price Kr. 9.

In the first part of his thesis, the author investigates stability of certain linear operators,

$$L = \rho(E) - h^r (d^r/dx^r)\sigma(E) + h^{r+1} (d^{r+1}/dx^{r+1})\tau(E), \tag{1}$$

associated with numerical integration formulas for sets of differential equations of the form

$$d^r\bar{y}/dx^r = \bar{f}(x, \bar{y}), \tag{2}$$

where $\bar{y}$ and $\bar{f}$ are s-dimensional vectors, $\rho(\zeta)$, $\sigma(\zeta)$, and $\tau(\zeta)$ are polynomials of degree k with real coefficients, and E is the displacement operator defined by $Eu(x) = u(x + h)$, for any function $u(x)$. The number k is called the order of L.

There exists an integer, p, called the degree of L, such that $Lu(x) = O(h^{p+r})$ as $h \to 0$. L is said to be stable if all zeros of $\rho(\zeta)$ are of modulus $\leqq 1$, and the zeros with modulus 1 are of multiplicity $\leqq r$.

The author shows that stability of L is equivalent to "stable convergence" of certain solutions of the difference equation associated with L to the solution of equation (2). If L is unstable the numerical solutions are "strongly unstable," and the integration formula is practically useless. Several theorems are given concerning the largest possible degree of stable operators of a given order. For example, if $\tau(\zeta) \equiv 0$, L is unstable if $p > 2([k/2] + [(r+1)/2])$; here $[x]$ denotes the largest integer $\leqq x$.

If the initial-value problem, $dy/dx = qy$, $y(0) = 1$, q constant, is treated numerically with a stable operator L of order k and degree $k + 2$, where $\tau(\zeta) \equiv 0$, the solution is a linear combination of basic solutions, ζ_{jh}^n, $1 \leqq j \leqq k$, $n = 0, 1, 2, \cdots$, where ζ_{jh} are the roots of the characteristic polynomial, $\rho(\zeta) - qh\sigma(\zeta)$. Let ζ_j be the roots of $\rho(\zeta)$; in the present case, $|\zeta_j| = 1$ and ζ_j is single for all j. The author shows that, for $h \to 0$, $\zeta_{jh} \sim \zeta_j(1 + k_j qh)$, where the $k_j = \sigma(\zeta_j)/[\zeta_j \rho'(\zeta_j)]$ are called growth parameters and are real numbers, then $\zeta_{jh}^n \sim \zeta_j^n \exp(k_j qhn)$. One of the ζ_j, say ζ_1, is equal to 1, and $k_1 = 1$.

If $Re(q) < 0$, the solution $\zeta_{1,h}^n \sim e^{qhn}$ decays, but may be dominated by some of the other basic solutions which increase or decrease more slowly. If there exist solutions which increase for $Re(q) < 0$, the operator L is called weakly unstable; this occurs if at least one of the k_j, $j \geqq 2$, is <0. It is shown that a stable operator of even order k and maximum degree $k + 2$ is weakly unstable; such an operator generates an oscillatory solution whose amplitude increases at least as rapidly as $|\exp(-qnh/3)|$ if $Re(q) < 0$.

In the second part of this thesis the author is concerned with estimating the norm of the error vector for $r = 1$. First, the error is evaluated for the linear variational system $d\bar{z}/dx = B(x)\bar{z}$ associated with (2), where $B(x)$ is the Jacobian, $(\partial\bar{f}/\partial\bar{y})_{\bar{y}=\bar{y}(x)}$. Then the effect of the linearization is estimated separately. Three error formulas are given. The second one, involving the directional derivative, $\mu[B(x)] = \lim_{\lambda\to 0+} \lambda^{-1}[\| I + \lambda B(x) \| - 1]$, yields particularly good results if the numerical solution is smooth and if $\mu[B(x)] < 0$.

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81[X].—A KORGANOFF, with the collaboration of L. BOSSETT, J. L. GROBOILLOT & J. JOHNSON, *Méthodes de Calcul Numérique*, Tome I: *Algèbre non linéaire*, Dunod, Paris, 1961, xxvii + 375 p., 25 cm. Price 58 NF.

The volume being reviewed is the first of a projected series. The characteristic-value problem is included, but matrix inversion is not, except to the extent that an iterative method that applies to a system of nonlinear equations would apply also to a system of linear equations.

The French literature on numerical analysis is sparse indeed, and hitherto has been but slightly affected by the advent of the electronic computer. This book goes far toward filling the gap there, and would be a substantial contribution to the literature in any language.