The Compositeness Of The Thirteenth Fermat Number

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Fermat numbers are numbers of the form $F_n = 2^{2^n} + 1$. As is well known, these numbers are prime for $n = 0, 1, 2, 3$, and 4. In fact, Fermat conjectured that they all were prime. Since then, however, factors of many F_n for $n > 4$ have been discovered [4]. In particular, factors are known for $n = 5, 6, 9, 10, 11, 12, 15, 16, 18, 23, 36, 38, 39, 55, 58, 63, 73, 77, 81, 117, 125, 144, 150, 207, 226, 228, 250, 267, 268, 284, 316, 452$, and 1945. On the other hand, Morehead and Western [1], [2] showed over fifty years ago that F_7 and F_8 are composite. Thus, F_{13} was the first number whose character was unknown, although [4] it has no factors less than 2^{35}.

Since F_n is prime if and only if

$$3^{(F_n-1)/2} \equiv -1 \pmod{F_n},$$

a program was written for the IBM 7090 to check this criterion and determine the character of F_{13}. At the beginning of December 1960, the computation was made and the result shows that F_{13} is composite.

The computation required 6 hours 17 minutes of machine time, including time to punch a number of intermediate residues. The result was checked by a rerun on a different day and all residues were identical. Moreover, as a check on the program, Morehead's result [1] for F_7, Morehead and Western's result [2] for F_8, and Robinson's result [3] for F_{10} were obtained.

The calculation to determine the character of F_{14} requires about $49\frac{1}{2}$ hours. This and the recalculation to check the result are each about half finished. However, lack of available machine time has temporarily suspended the computation.

When machine time does become available, in addition to finishing the computation for F_{14}, a search will be made to insure that Robinson's list [4] of factors p of Fermat numbers contains all $p < 2^{35}$. At present it contains all $p < 2^{32}$ and all $p < 2^{35}$ for which $p \equiv 1 \pmod{2^{15}}$. It should be noted that this last search could yield new factors only for $7 \leqq n \leqq 12$ since all factors of F_n are of the form $k \cdot 2^{n+2} + 1$.

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1. J. C. Morehead, "Note on Fermat's numbers," *Bull. Amer. Math. Soc.*, v. 11, 1905, p. 543–545.
2. J. C. Morehead & A. E. Western, "Note on Fermat's numbers," ibid, v. 16, 1909, p. 1–6.
3. R. M. Robinson, "Mersenne and Fermat numbers," *Proc. Amer. Math. Soc.*, v. 5, 1954, p. 842–846.
4. R. M. Robinson, "A report on primes of the form $k \cdot 2^n + 1$ and on factors of Fermat numbers," ibid., v. 9, 1958, p. 673–681.

Received January 18, 1961; revised April 25, 1961.