88[A–E, G, I, K, L, M, X].—GRANINO A. KORN & THERESA M. KORN, *Mathematical Handbook for Scientists and Engineers*, McGraw-Hill Book Co., New York, 1961, xiv + 943 p., 24 cm. Price $20.00.

Workers in every walk of mathematical life will gratefully welcome this latest addition to an illustrious series of Handbooks. In its variety and scope it may well be the largest collection of widely useful mathematical facts and data ever compiled. It admirably fills a too-long existing gap among the handbooks available to workers in technical fields. As a "tool of the trade" its price is certainly reasonable, probably offering the "lowest cost per (mathematical) fact."

"This handbook is intended, first, as a comprehensive reference collection of mathematical definitions, theorems, and formulas for scientists, engineers, and students. Subjects of both undergraduate and graduate level are included. The omission of all proofs and the concise tabular presentation of related formulas have made it possible to incorporate a relatively large amount of reference material in one volume.

"The handbook is, however, not intended for reference purposes alone; it attempts to present a connected survey of mathematical methods useful beyond specialized applications. Each chapter is arranged so as to permit rapid review of an entire mathematical subject," and chapter introductions, notes, and cross-references interrelate the many topics "for a broad view of the entire field of mathematics."

To meet the requirements of different readers the material has been arranged at three levels:

"1. The most important formulas and definitions have been collected in tables and boxed groups permitting rapid reference and review.

"2. The main text presents, in large print, a concise, connected review of each subject.

"3. More detailed discussions and advanced topics are presented in small print."

The following summary of the book's twenty-one chapters and appendices gives a brief indication of the scope of the material.

Chapters 1 through 5 review the basic college material on algebra, analytic geometry (plane and solid), elementary and advanced calculus, including Lebesgue and Stieltjes integrals, and vector analysis. Chapters 6, 7, and 8 cover curvilinear coordinates, functions of a complex variable, and Laplace and other integral transformations, respectively. Chapters 9 through 11 cover ordinary and partial differential equations (including transform methods, method of characteristics), and maxima and minima, including the calculus of variations.

Chapters 12 through 14 deal with various aspects of mathematical models. Chapter 12 introduces the elements of modern abstract language and covers concepts such as groups, fields, topological spaces, and Boolean algebras. Chapter 13 deals with matrices, and quadratic and hermitian forms. Chapter 14 treats linear vector spaces and transformations, including matrix representation, eigenvalues,

and group representations. Chapter 15 handles the subject of linear integral equations, boundary-value problems, and eigenvalue problems. Chapters 16 and 17 give a good outline of the related subjects of tensor analysis and differential geometry.

Chapters 18 and 19 recognize the increasing importance of statistical methods in many fields and provide over 100 pages devoted to probability and random processes, and mathematical statistics. The material is given in appealing detail, and the modern worker has the comfort of finding succinctly in a single source many of the not always easy-to-find formulas and results on such topics as multi-dimensional distributions, limit theorems, generalized Fourier analysis including correlation and power spectra, sampling distributions, and statistical estimation and testing of hypotheses.

Chapter 20, on numerical calculations and finite differences, reviews the standard methods and has a section on difference equations. Included are numerical methods for matrix inversion, eigenvalues, interpolation and approximation, ordinary and partial differential equations, and numerical harmonic analysis, among others. Chapter 21 is essentially a brief collection of formulas on the properties of elementary and higher transcendental functions.

A significant portion (about one-sixth) of the volume consists of six appendices as follows: formulas for plane figures and solids; plane and spherical trigonometry; permutations, combinations, and related topics; tables of Fourier expansions and Laplace-transform pairs; tables of indefinite and definite integrals; and twenty numerical tables.

The book is rounded out with a glossary of symbols and notation showing where each item is explained, and a comprehensive index of almost thirty pages that enables the book to be used as a mathematical dictionary.

The painstaking care with which each subject is organized is shown by effective use of summary tables and boxes. The box is a device in common use abroad, which might well be used more widely here. A small sampling of the tables and boxes will be helpful to the prospective user and give an insight into the valuable nature of the material: a table of formulas dealing with tangents, normals, and polars for each of the four classes of conic sections; a box showing various forms for the equation of a plane, and line, in both cartesian and vector notation; a table of properties of Fourier transforms—linearity, change of scale, shift, convolution, modulation, differentiation, Parseval's theorem; tables of operations on scalar and vector point functions; tables relating to a wide variety of transformations and other properties of the various orthogonal curvilinear coordinate systems; a table of real and imaginary parts, zeros, and singularities of common functions; a graphical set of sixty conformal mappings of regions in the complex plane; a table of definitions for different types of tensors; boxes with definitions of Riemann space and associated quantities such as covariant derivatives, Christoffel three-index symbols, and the first and second fundamental quadratic differential forms of a surface; á table explaining fourteen numerical parameters describing properties of one-dimensional probability distributions; boxed formulas for moments, characteristic and other generating functions for one-dimensional distributions, and for two- and more-dimensional probability and marginal distributions; tables for the many formulas and properties of a dozen or more discrete and continuous distributions

of most importance in applications, including the features "typical interpretation" and "approximations," which are rarely presented in standard treatments in such useful form; tables of formulas for tests of hypotheses and confidence intervals for normal populations; a box for the unit-step functions, with accompanying sketches, and of relations involving the delta function and its "derivatives"; and a table and sketches of various types of pulses and waveforms and their characteristics.

In addition to the appended numerical tables there are several short tables in the chapter on numerical calculations: 5- to 7-place tables for Lagrange, Newton, Stirling, Bessel, Everett, and Steffensen interpolation, and tables for abscissas and weights for Gauss and Chebyshev quadrature formulas.

While not detracting materially from the excellence of the book, mention of a few necessary corrections that were noted may be of help to the user. The pagination for Chapter 6 should be corrected in the Table of Contents as follows: Sections 6.4, 6.5, 6.6 begin on pages 170, 173, and 173, respectively; on page 112 the symbol "$\rightarrow 0$" is omitted under "lim" on the right-hand side of equation (4.6–43); on page 443, in the displayed equation at the top of the page, "$= 0$" should be replaced by "$\neq 0$"; on page 489, the second line under the last box, Sec. 5.10–3 should read Sec. 16.10–3; on page 566, the typography is confusing in the last line of the table owing to wrong size of type—the mathematical expressions should read

$$\frac{1}{\beta^{\alpha}\Gamma(\alpha)}\, x^{\alpha-1}e^{-x/\beta} \quad \text{and} \quad \frac{1}{\Gamma(\alpha)}\, \Gamma_{x/\beta}(\alpha);$$

in the same table, the last two entries in the column "Characteristic function" are known in explicit form and should be given, namely, $F(\alpha;\ \alpha + \beta;\ it)$ (confluent hypergeometric function) and $(1 - \beta it)^{-\alpha}$; on page 570, in equation (18.8–29) the minus sign is omitted from the exponential, and in equation (18.8–30) the multiplier $(1/\pi\alpha)$ is omitted from the expression for $\phi_x(x)$; on page 626 the reference in the heading of Sec. 19.8–2 should be "Sec. 18.12–2" instead of "Sec. 18.11–2"; on page 935, the index entry "Probability distribution" might well have included a reference to Sec. 18.8, as it has eight tables showing valuable information about the most important special distributions in statistics.

It may be appropriate to mention several additional matters that may be of value in connection with any later edition. The subject of random numbers apparently is not included anywhere in the book, and it would seem that at least one of the most important modern works on this topic warrants mention either in Chapter 18, on Probability, or in Chapter 19, on Mathematical Statistics, namely, The Rand Corporation's *A Million Random Digits with 100,000 Normal Deviates*, The Free Press Publications, Glencoe, Illinois, 1955. Also, some of the older references listed at the end of Chapter 19 should be replaced by their more modern versions; for example, Arkin and Colton is in a fourth, revised edition (1955), and P. G. Hoel is in a second edition (1954). In addition, the following works might be included as being very useful for reference and application purposes:

Oscar Krisen Buros, *Statistical Methodology Reviews*, 1941–50, John Wiley & Sons, Inc., New York, 1951;

M. G. Kendall & W. R. Buckland, *A Dictionary of Statistical Terms*, Hafner Publishing Co., New York, 1957;

E. P. Adams & R. L. Hippisley, *Smithsonian Mathematical Formulae and*

Tables of Elliptic Functions, Smithsonian Miscellaneous Collections, Vol. 74, No. 1, Washington, D. C., 1922 (or later edition);

Statistics Manual, NAVORD Report 3369, Naval Ordnance Test Station, China Lake, California, 1955.

Several minor points may be noted with regard to the numerical tables in Appendix F. To the eleven numerical constants listed should be added Euler's constant, which occurs in a number of places in the text. There is space for increasing the number of decimal places shown to at least 10; this should be done to increase their usefulness. The typographical layout for several of the tables is hard on the eyes because little or no space is allowed between entries in adjacent columns. The columnar lines alone do not provide effective separation, so that the entries running across the page merge into one another. This applies to all or part of the tables for squares, integral sine and cosine, χ^2 distribution, and F distribution. This can be remedied either by use of smaller type or by printing the tables along the length rather than the width of the page, as is done with some of the other tables, resulting in much greater legibility.

Much of the material of the book is necessarily gathered from other sources. In a number of places, especially the figures, the source is cited from among the references at the end of the chapter. It would be helpful if such citation (admittedly laborious) could be done more systematically, as this could save a great deal of time and effort spent in searching through the listed references in order to follow up a particular theorem or development.

As regards the physical aspects, one would wish that a book of such utility could be constructed in such a manner as to better be able to withstand the great amount of handling it is bound to receive, perhaps by being issued in the almost indestructible form achieved by the binders used in the tax and accounting services.

Even with the minor shortcomings indicated here, this mathematical handbook is of such unique value that it can be unhesitatingly recommended for the intimate possession of everyone with a serious interest in the theory or application of virtually any aspect of mathematics.

JULIUS LIEBLEIN

Applied Mathematics Laboratory
David Taylor Model Basin
Washington 7, D. C.

89 [**E, L**].—L. N. NOSOVA, *Tablit͡sy funkt͡siĭ Tomsona i ik͡h pervyk͡h proizvodnyk͡h* (*Tables of Thomson Functions and their First Derivatives*), Izdatel'stvo Akademiĭ Nauk SSSR, Moscow, 1960, 422 p., 27 cm. Price 49 Rubles.

This new addition to the series of tables prepared at the Computation Center of the Academy of Sciences, USSR, consists of two main tables. The first of these presents values of the Thomson (or Kelvin) functions ber x, bei x, ker x, kei x, and of their first derivatives to 7S for $x = 0(.01)10$. The second principal table gives values of modified functions consisting of the Kelvin functions of the first kind (ber, bei) and their first derivatives, each multiplied by $e^{-x/\sqrt{2}}$, and the functions of the second kind (ker, kei) and their first derivatives, each multiplied by $e^{x/\sqrt{2}}$. These data are also given to 7S, for $x = 10(.01)100$. Corresponding values of $e^{x/\sqrt{2}}$ are tabulated in an adjoining column to 7S. The entries throughout appear as 7-digit integers multiplied by an appropriate power of 10.