

availability at a reasonable cost should be welcomed by all students of numerical analysis.

H. P.

- 34 [K].—W. S. CONNOR & SHIRLEY YOUNG, *Fractional Factorial Designs for Experiments with Factors at Two and Three Levels*, NBS Applied Mathematics Series, No. 58, National Bureau of Standards, Washington, D. C., 1961, v + 65 p. Price \$0.40.

This publication contains a collection of fractional factorial designs for experiments in which some factors are to be studied at two levels or conditions and others at three levels. It is the sequel to two other catalogs [1], [2] of designs in the National Bureau of Standards Applied Mathematics Series that contain, respectively, plans for m factors each at two levels, and plans for n factors each at three levels. This new document gives plans for the mixed series involving $(m + n)$ factors, where the m factors each at two levels and the n factors each at three levels are given for 39 combinations ($2^m 3^n$) of positive integer values of m and n for which $5 \leq m + n \leq 10$.

For each design the following are given: number of effects estimated, number of treatment combinations employed in the design, fraction of complete factorial experimental plan, analysis, and construction.

The method of construction of designs is described in Section 2. Fractions are selected so that low-order interaction effects, including main effects, are aliased with each other as little as possible. Section 3 contains a description of the mathematical model, in which it is assumed that all interactions between three or more factors are nonexistent, and a procedure for estimating the parameters contained in the model. Section 4 contains a discussion of procedures to test hypotheses and to construct confidence intervals. A worked example of $2^3 3^2$ design is presented in Section 5.

Section 6 is devoted to six particular designs for which the interaction effects between factors at three levels are defined in a different manner from that of the other designs.

H. H. KU

National Bureau of Standards
Washington, D. C.

1. NATIONAL BUREAU OF STANDARDS, *Fractional Factorial Experiment Designs for Factors at Two Levels*, NBS Applied Mathematics Series, No. 48, U. S. Gov. Printing Office, Washington, D. C., 1957.

2. W. S. CONNOR & MARVIN ZELEN, *Fractional Factorial Experiment Designs for Factors at Three Levels*, NBS Applied Mathematics Series, No. 54, U. S. Gov. Printing Office, Washington, D. C., 1959.

- 35 [K].—N. V. SMIRNOV, Editor, *Tables for the Distribution and Density Functions of t -Distribution ("Student's" Distribution)*, Pergamon Press Ltd., New York, 1961, 130 p., 28 cm. Price \$12.50.

This book, which is Volume 16 in the Mathematical Tables Series of Pergamon Press, is a translation of the Russian work issued by the V. A. Steklov Mathematical Institute of the Academy of Sciences of the U. S. S. R. There are three main

tables of the (cumulative) distribution and density function of the ordinary (central) t -distribution presented to six decimal places for several ranges of the argument and parameter, followed by four auxiliary tables and one table of interpolation coefficients.

The density function of the t -distribution is

$$s_\nu(t) = K_\nu \cdot \left(1 + \frac{t^2}{\nu}\right)^{-\frac{\nu+1}{2}}$$

where

$$K_\nu = (\pi\nu)^{-1/2} \Gamma[\frac{1}{2}(\nu + 1)] / \Gamma(\frac{1}{2}\nu);$$

and the distribution function is

$$S_\nu(t) = \int_{-\infty}^t s(u) du.$$

Table I gives, in parallel columns, the values of $S_\nu(t)$ and $s_\nu(t)$ for $t = 0.00(0.01)3.00(0.02)4.50(0.05)6.50$, with $\nu = 1(1)12$, and for almost as many values of t with $\nu = 13(1)24$. Some of the printing is imperfect, one of the values, namely, $S_{21}(1.00)$ on page 6, having its fourth decimal place almost completely missing. Fortunately, this value of t is one of those repeated at the top of the page following, so that the missing digit, 6, is available.

Table II gives the distribution function $S_\nu(t)$ for large values of the argument, namely, $t = 6.5(1.0)9.0$, and $\nu = 1(1)10$.

Table III gives $S_\nu(t)$ for large numbers of degrees of freedom for $t = 0.00(0.01)2.50(0.02)3.50(0.05)5.00$, namely, for $\nu = 25(1)35$. The first two differences of the function are also given in convenient form alongside the functional values.

Table IV lists the values of $S_\nu(t)$, according to the values of $\xi = 1000/\nu$, for $t = 0.00(0.01)2.50(0.02)5.00$ (the limit 5.00 is omitted in the Contents and Introduction) for $\xi = 30(2)0$. The values of the ξ -entry are equivalent to the values $\nu = 33\frac{1}{3}, 35\frac{2}{7}$, (etc.), 250, 500, ∞ (corresponding to the normal distribution $\Phi(t)$). This type of tabulation facilitates harmonic interpolation, which is necessary for large values of ν .

Table V gives values of the polynomials $C_i(t)$, $i = 1$ to 4, that occur in R. A. Fisher's formula for the distribution function in inverse powers of ν ,

$$S_\nu(t) = \Phi(t) - \sum_{i=1}^{\infty} \frac{C_i(t)}{\nu^i},$$

for $t = 0.00(0.01)5.00$. These values, together with those of $C_5(t)$ that are given for a small number of values of t , were used in the calculation of Table III.

Table VI gives the values of K_ν and its common logarithm for $\nu = 1(1)24$. In addition, two other quantities, p_s and q_s , are tabulated for $s = 2(1)21$, for which no explanation is given in the Introduction or anywhere else in the volume. Apparently $p_s = 1/\sqrt{s}$, but the reviewer was unable to determine the meaning of q_s or the reason for tabulating these quantities.

Table VII gives the upper percentiles of the t -distribution corresponding to the probability levels $Q = .4, .25, .1, .05, .025, .01, .005, .0025, .001, .0005$, for $\nu = 1(1)30$ and various other values to 10,000, and ∞ .

Table VIII gives, in concise form, the coefficient $B = k(1 - k)/4$ in Bessel's quadratic interpolation formula. By showing only the values of k for which the value of B changes in the third decimal place, the table is cut down to little more than one-tenth of the size otherwise necessary.

The introductory part of the volume includes brief sections on interpolation, with numerical examples, formulas for calculating and checking the tables, and a list of eight applications. It would have been very helpful to give examples of applications listed, along the lines of the Introduction to *Biometrika Tables for Statisticians*, Vol. I, for example, and to indicate needs for tabulations of such detail and accuracy. Several key references besides the one to Fisher in *Metron* would also have been helpful.

As already indicated, the quality of printing is less than perfect. The alignment of the columns is poor, making it unnecessarily difficult to read across the page to find the entry corresponding to a t argument. It might be pointed out that provision of a column for t at the right of each page as well as the one at the left would largely have alleviated this difficulty. Other minor shortcomings are lack of a heading at the top of each page to identify the table at a glance; omission of the subscript ν in $S(t)$ at the bottom of page 7 and inconsistency in showing the argument values, as in $t = 0(0.01)5.00$; writing $C(t)$ for $C_5(t)$ at the top of page 8, where also the reference to values of the parameter ν is irrelevant; omission of the prime in the derivative $S'(t)$ at the beginning of Section III on page 11; and erroneously writing the t -interval for Table II as 1.0 instead of 0.1 in both the table of contents and in the title on page 69. In Table III on page 75 the subscripts on the differences Δ should be written as exponents (the bar in $\bar{\Delta}^2$ means that the second differences are all negative).

In spite of such shortcomings, this volume represents the most detailed tabulation of Student's t -distribution available and, while it is not recommended for the general practitioner, will be indispensable to statisticians and others who require finely tabulated values for theoretical or other reasons.

J. LIEBLEIN

Applied Mathematics Laboratory
David Taylor Model Basin
Washington 7, D. C.

36 [L].—E. N. DEKANOSIDZE, *Tables of Lommel's Functions of Two Variables*, Pergamon Press, New York, 1960. 492 p. Price \$20.00.

This is an English translation from the Russian. The original was reviewed in *MTAC*, v. 12, 1958, p. 239–240. The introduction contains several typographical errors, and these have been noted in a recent paper by J. Boersma "On the computation of Lommel's functions of two variables," in *Mathematics of Computation*, v. 16, 1962, p. 232.

Y. L. L.