

- 4 [H].—HENRY C. THACHER, JR., *Real Roots of the Equation $x \tan y + \tanh y = 0$* , ms. of six leaves deposited in UMT File.

The author describes in detail the procedure he followed in the preparation of these original manuscript tables, which give to six decimal places the first two real roots of the transcendental equation $x \tan y + \tanh y = 0$ for $x = 0(.05)1$ and for $\pm x^{-1} = 1(-.05)0$, respectively, and the first three real roots of that equation for $-x = 0(.05)1$, together with an auxiliary table to facilitate interpolation.

Such data are used, according to the author, in the application of one theory of the convective heat transfer between parallel plates. (Reference is made to A. F. Lietzke, *Theoretical and Experimental Investigation of Heat Transfer by Laminar Natural Convection between Parallel Plates*, NACA Report 1223, 1955.)

J. W. W.

- 5 [I, K, P].—B. M. BROWN, *The Mathematical Theory of Linear Systems*, John Wiley & Sons, Inc., New York, 1961, x + 267 p., 9 cm. Price \$8.00.

For its size this book is amazingly broad and thorough in its coverage of linear systems. Since the author is British, however, the manner of expression and sometimes the notation may seem strange to the American reader.

After a general discussion of linear differential equations, operational methods of solution are introduced. The unilateral Laplace transform is then used to solve equations with given initial conditions. Fourier series and integrals and the bilateral Laplace transform are then brought in, along with impulse, step, and ramp functions.

Linear systems are then discussed with the aid of block diagrams and simple physical examples, which are solved by various mathematical methods, including the weighting function. Feedback is then introduced, along with the stability criteria of Routh, Hurwitz, and Nyquist. The graphical methods using M -circles and root loci are summarized.

Statistical methods are used to introduce the concepts of correlation functions and spectral density, which are then applied to system optimization both with and without constraints.

After a short treatment of difference equations, the Z -transform is developed, followed by a treatment of sampling servos. Other topics treated include time-variant systems, multivariable systems, and interpolation systems.

The several appendices contain mathematical background materials and proofs. The book is probably too condensed to be a good college text, but it presents a good logical development and can serve as a valuable reference.

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- 6 [I, X].—D. S. MITRINOVIĆ & R. S. MITRINOVIĆ, "Sur une classe de nombres se rattachant aux nombres de Stirling," *Publ. Fac. Élect. Univ. Belgrade* (Série: Math. et Phys.), No. 60, 1961, 63 p. (French with Serbian summary.)

The two tables given in this publication are extensions of earlier ones by the same authors (see *Math. Comp.*, v. 15, 1961, p. 107; v. 16, 1962, p. 252).

Table I (p. 17–44) gives exact values of $(-)^m C_m^k$ for $k = 0(1)59$ and $m = 51(1)60$, with $k < m$. The values are integers having between 67 and 111 digits.

Table II (p. 45–62) gives exact values of the Stirling numbers of the first kind, $(-)^m S_n^{n-m}$, for $m = 7(1)59$ and $n = 51(1)60$, with $m < n$, and also for $m = 60$, $n = 61$. The values are integers having between 18 and 82 digits. Various values of S_n^{n-m} given in the tables were checked in laboratories at Liverpool, Rome, and Hamburg. In addition, the value of S_{70}^{10} was computed at Liverpool and Rome, and the 98-digit result is given separately on p. 8.

A. F.

7 [J].—I. J. SCHWATT, *An Introduction to the Operations with Series*, Chelsea Publishing Company, New York, 1961, x + 287 p., 21 cm. Price \$3.95.

The present, second edition is a reprint, with corrections, of the first edition of 1924, and this date must be kept in mind. Even so, the volume has several weaknesses, as considerable information on transcendental functions and related topics available in the literature of 1924 apparently was not known by the author, or if known, there are no references. For example, *Modern Analysis* by E. T. Whittaker and G. N. Watson, and *A Textbook of Algebra* by G. Chrystal are not mentioned. The preface states that the “book had its inception in the author’s efforts to obtain the value for the sum of the series of powers of natural numbers, in an explicit form and without the use of Bernoulli numbers. This problem led to the study of the higher derivatives of functions of functions, which in turn required certain principles in operations with series, which had to be established. By means of these and other principles, methods for the expansion of certain functions and the summation of various types of series were devised and other topics developed.” The volume is replete with numerous examples, and a number of ingenious devices are used. In the following we give some of the highlights of each chapter, along with comments which should enhance the usefulness of the volume.

Chapter I deals with derivatives and expansions in powers of x of $(\sum_{m=0}^r a_m x^m)^p$. Power series for $\sin^p x$, $\cos^p x$, $\tan^p x$, and their reciprocals are given in Ch. II for $p = 1$, and in Ch. IV for a general integer p . That the coefficients of powers of x in the expansions of $\tan x$ and $\sec x$ are related to the Bernoulli and Euler numbers, respectively, is not mentioned in Ch. II. These numbers are studied in Ch. XV. The operator δ^n , where $\delta = xd/dx$ is the subject of Ch. V. The operator is used to find the differential equation satisfied by certain series. If the solution to the differential equation can be found in a simple form, then the given series is summed. We recognize that the differential equation satisfied by the generalized hypergeometric function ${}_pF_q$ can be easily expressed with the operator δ . (See A. Erdélyi, W. Magnus, F. Oberhettinger, and F. G. Tricomi, *Higher Transcendental Functions*, v. 1, McGraw-Hill, 1953.) Indeed, many of the problems treated are of hypergeometric type. For example, $S = \sum_{n=1}^{\infty} (-)^{n-1} n^p x^n = \delta^p(x/x + 1)$. Extension of the results in Ch. V are given in Ch. X, and that technique is applied to sum trigonometrical series in Ch. XII. In Ch. VI, derivatives of continued products $\prod_{k=1}^n f(x, k)$ are treated for $f(x, k) = \sin kx$, $x + k$, and $1 - x^k$. Chapter VII generalizes some of the results in Chs. I, IV, and V. Separation of fractions into partial fractions is treated in Ch. VIII, and these results are used in Ch. IX to