Table I (p. 17–44) gives exact values of $(-)^m C_m^k$ for $k = 0(1)59$ and $m = 51(1)60$, with $k < m$. The values are integers having between 67 and 111 digits.

Table II (p. 45–62) gives exact values of the Stirling numbers of the first kind, $(-)^m S_n^{n-m}$, for $m = 7(1)59$ and $n = 51(1)60$, with $m < n$, and also for $m = 60$, $n = 61$. The values are integers having between 18 and 82 digits. Various values of S_n^{n-m} given in the tables were checked in laboratories at Liverpool, Rome, and Hamburg. In addition, the value of S_{70}^{10} was computed at Liverpool and Rome, and the 98-digit result is given separately on p. 8.

A. F.

7 [J].—I. J. SCHWATT, *An Introduction to the Operations with Series*, Chelsea Publishing Company, New York, 1961, x + 287 p., 21 cm. Price $3.95.

The present, second edition is a reprint, with corrections, of the first edition of 1924, and this date must be kept in mind. Even so, the volume has several weaknesses, as considerable information on transcendental functions and related topics available in the literature of 1924 apparently was not known by the author, or if known, there are no references. For example, *Modern Analysis* by E. T. Whittaker and G. N. Watson, and *A Textbook of Algebra* by G. Chrystal are not mentioned. The preface states that the "book had its inception in the author's efforts to obtain the value for the sum of the series of powers of natural numbers, in an explicit form and without the use of Bernoulli numbers. This problem led to the study of the higher derivatives of functions of functions, which in turn required certain principles in operations with series, which had to be established. By means of these and other principles, methods for the expansion of certain functions and the summation of various types of series were devised and other topics developed." The volume is replete with numerous examples, and a number of ingenious devices are used. In the following we give some of the highlights of each chapter, along with comments which should enhance the usefulness of the volume.

Chapter I deals with derivatives and expansions in powers of x of $(\sum_{m=0}^{r} a_m x^m)^p$. Power series for $\sin^p x$, $\cos^p x$, $\tan^p x$, and their reciprocals are given in Ch. II for $p = 1$, and in Ch. IV for a general integer p. That the coefficients of powers of x in the expansions of $\tan x$ and $\sec x$ are related to the Bernoulli and Euler numbers, respectively, is not mentioned in Ch. II. These numbers are studied in Ch. XV. The operator δ^n, where $\delta = xd/dx$ is the subject of Ch. V. The operator is used to find the differential equation satisfied by certain series. If the solution to the differential equation can be found in a simple form, then the given series is summed. We recognize that the differential equation satisfied by the generalized hypergeometric function ${}_pF_q$ can be easily expressed with the operator δ. (See A. Erdélyi, W. Magnus, F. Oberhettinger, and F. G. Tricomi, *Higher Transcendental Functions*, v. 1, McGraw-Hill, 1953.) Indeed, many of the problems treated are of hypergeometric type. For example, $S = \sum_{n=1}^{\infty}(-)^{n-1} n^p x^n = \delta^p(x/x + 1)$. Extension of the results in Ch. V are given in Ch. X, and that technique is applied to sum trigonometrical series in Ch. XII. In Ch. VI, derivatives of continued products $\prod_{k=1}^{n} f(x, k)$ are treated for $f(x, k) = \sin kx$, $x + k$, and $1 - x^k$. Chapter VII generalizes some of the results in Chs. I, IV, and V. Separation of fractions into partial fractions is treated in Ch. VIII, and these results are used in Ch. IX to evaluate $\int \frac{x^m\,dx}{x^n \pm 1}$ where m and n are integers. The author studies

$$S = \sum_{n=0}^{\infty} \prod_{k=0}^{n} \left(\frac{a+k}{b+k}\right) r^n,$$

and shows that it can be written as a finite sum if $b - a$ is a positive integer. The value of S for $r = 1$ is determined. In the course of this discussion, the beta integral is evaluated. The author does not seem to recognize that $S = (a/b)\,{}_2F_1(1, a + 1; b + 1; r)$ where ${}_2F_1$ is the Gaussian hypergeometric function and

$$S = \frac{\Gamma(b)}{\Gamma(a)\Gamma(b-a)} \int_0^1 t^a(1-t)^{b-a-1}(1-tr)^{-1}\,dt,$$

provided $R(b + 1) > R(a + 1) > 0$ and $|\arg(1 - r)| < \pi$. Chapter XI takes up the separation in partial fractions of trigonometric expressions such as $\cos^p x/\cos nx$, $p < n$.

Chapter XIII is concerned with the evaluation of definite integrals such as $\int_a^b x^{-1}e^x\,dx$. In an aside, the author says (p. 244), "If a personal reference be permitted, the author has spent considerable effort in trying to express in terms of elementary functions $S = \sum_{n=1}^{\infty} \frac{x^n}{n!n}$ and the solution of $x\,d^2S/dx^2 + (1 - x)\,dS/dx = 1$ which is satisfied by S. It is hoped that mathematicians will feel induced to take up this and similar problems in the operation with series which are waiting solution and which have such an important bearing on mathematical analysis." Now $S = -\gamma - \ln x - \int_{-x}^{\infty} t^{-1}e^{-t}\,dt$, and Liouville showed that this could not be expressed in terms of elementary functions. In this connection, see *Integration in Finite Terms*, by J. F. Ritt, Columbia Univ. Press, 1948, p. 49.

The study of sums of some conditionally convergent series under rearrangement is taken up in Ch. XIV. Examples include $\sum_{k=0}^{\infty} (-)^k(b + kh)^{-1}$ rearranged so that m positive terms are followed by n negative terms.

In conclusion, the volume contains many results of interest in applied work, but the reader is cautioned to keep in mind the developments of the last four decades.

Y. L. L.

8 [K].—E. L. PETERSON, *Statistical Analysis and Optimization of Systems*, John Wiley & Sons, 1961, xi + 190 p., 23 cm. Price $9.75.

The title of the book may suggest that this book is an extensive treatise on statistical analysis and optimization of systems. However, this is not the case here. The book treats mainly time-varying linear systems such as missiles and fire control systems. The first three chapters and the first two sections of Chapter 4 (Chapter 1, Linear System Theory; Chapter 2, Statistics of Random Variable; Chapter 3, Response to Distributed Inputs; and Chapter 4, Systems Analysis and Design—General Approach and the Adjoint Method of Analysis) are not too rigorous summaries of some transform techniques, stability theory, theory of probabilities and some filtering and prediction theory, as can be found in a standard textbook such