

evaluate  $\int \frac{x^m dx}{x^n \pm 1}$  where  $m$  and  $n$  are integers. The author studies

$$S = \sum_{n=0}^{\infty} \prod_{k=0}^n \left( \frac{a+k}{b+k} \right) r^n,$$

and shows that it can be written as a finite sum if  $b - a$  is a positive integer. The value of  $S$  for  $r = 1$  is determined. In the course of this discussion, the beta integral is evaluated. The author does not seem to recognize that  $S = (a/b)_2F_1(1, a + 1; b + 1; r)$  where  ${}_2F_1$  is the Gaussian hypergeometric function and

$$S = \frac{\Gamma(b)}{\Gamma(a)\Gamma(b-a)} \int_0^1 t^a (1-t)^{b-a-1} (1-tr)^{-1} dt,$$

provided  $R(b+1) > R(a+1) > 0$  and  $|\arg(1-r)| < \pi$ . Chapter XI takes up the separation in partial fractions of trigonometric expressions such as  $\cos^p x / \cos nx$ ,  $p < n$ .

Chapter XIII is concerned with the evaluation of definite integrals such as  $\int_a^b x^{-1} e^x dx$ . In an aside, the author says (p. 244), "If a personal reference be permitted, the author has spent considerable effort in trying to express in terms of elementary functions  $S = \sum_{n=1}^{\infty} \frac{x^n}{n!n}$  and the solution of  $x d^2S/dx^2 + (1-x) dS/dx = 1$  which is satisfied by  $S$ . It is hoped that mathematicians will feel induced to take up this and similar problems in the operation with series which are waiting solution and which have such an important bearing on mathematical analysis." Now  $S = -\gamma - \ln x - \int_x^{\infty} t^{-1} e^{-t} dt$ , and Liouville showed that this could not be expressed in terms of elementary functions. In this connection, see *Integration in Finite Terms*, by J. F. Ritt, Columbia Univ. Press, 1948, p. 49.

The study of sums of some conditionally convergent series under rearrangement is taken up in Ch. XIV. Examples include  $\sum_{k=0}^{\infty} (-)^k (b + kh)^{-1}$  rearranged so that  $m$  positive terms are followed by  $n$  negative terms.

In conclusion, the volume contains many results of interest in applied work, but the reader is cautioned to keep in mind the developments of the last four decades.

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**8 [K].**—E. L. PETERSON, *Statistical Analysis and Optimization of Systems*, John Wiley & Sons, 1961, xi + 190 p., 23 cm. Price \$9.75.

The title of the book may suggest that this book is an extensive treatise on statistical analysis and optimization of systems. However, this is not the case here. The book treats mainly time-varying linear systems such as missiles and fire control systems. The first three chapters and the first two sections of Chapter 4 (Chapter 1, Linear System Theory; Chapter 2, Statistics of Random Variable; Chapter 3, Response to Distributed Inputs; and Chapter 4, Systems Analysis and Design—General Approach and the Adjoint Method of Analysis) are not too rigorous summaries of some transform techniques, stability theory, theory of probabilities and some filtering and prediction theory, as can be found in a standard textbook such

as J. H. Laning, Jr. & R. H. Battin, *Random Process in Automatic Control*, McGraw-Hill, 1956. The merit of the book lies mainly in Chapters 5 and 6, Optimum Systems and Applications in Optimal Systems, where such things as determination of the optimum impulse response and the optimum system block diagram are discussed with the usual mean-square error criterion. Applications are taken from problems in fire control, tracking, prediction, guidance, and navigation. Publications by M. Shinbrot are referred to quite often.

In these chapters, analysis and synthesis processes of control systems are illustrated from a very rough, sketchy beginning to a more and more precise determination of various system parameters. The book occupies a middle position in the spectrum of technical books, in that it is not recommended for beginners and yet it is not very useful for people actually working in this area. However, students in control systems can benefit from such step-by-step explanations of synthesis processes.

On the whole, the reviewer feels that the book has not come up to the expectation raised in the reader's mind when he looks at the chapter headings.

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9 [L].—V. S. AIZENSHTAT, V. I. KRYLOV & A. S. METLESKII, *Tables for Calculating Laplace Transforms and Integrals of the form*  $\int_0^\infty x^s e^{-x} f(x) dx$ , Izdatel'stov Akad. Nauk SSSR, Minsk, 1962, 378 p. (Russian).

This book gives tables of Gaussian quadrature formulas for approximate evaluation of the integrals in the title. These formulas have the form

$$\int_0^\infty x^s e^{-x} f(x) dx = \sum_{k=1}^n A_k f(x_k)$$

where the  $A_k$  and  $x_k$  depend on the parameter  $s$  and the value of  $n$ . The  $A_k$  and  $x_k$  are chosen so that the approximation is exact whenever  $f(x)$  is a polynomial of degree  $\leq 2n - 1$ . This means that the  $x_k$  are the zeros of the  $n$ th degree generalized Laguerre polynomial  $L_n^{(s)}(x)$ , which satisfies the orthogonality condition

$$\int_0^\infty x^{s-x} L_n^{(s)}(x) P(x) dx = 0,$$

where  $P(x)$  is an arbitrary polynomial of degree  $\leq n - 1$ .

The first 22 pages of the book discuss properties of these formulas and give some examples of their use. The remainder of the book contains the formulas and is divided into three tables. Table 1 gives formulas for  $s = -0.90(0.02)0.00$ ; Table 2, formulas for  $s = 0.55(0.05)3.00$ ; and Table 3, formulas for  $s = -\frac{3}{4}, -\frac{1}{4}, n + \frac{k}{3}$ , where  $n = -1(1)2$ ,  $k = 1, 2$ . For each value of  $s$ , the numbers  $A_k$ ,  $x_k$ , and  $A_k e^{x_k}$  are given to 8 significant figures for  $n = 1(1)15$ .

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