

10 [L].—BURTON D. FRIED & SAMUEL D. CONTE, *The Plasma Dispersion Function: The Hilbert Transform of the Gaussian*, Academic Press, New York and London, 1961, 419 p., 26 cm. Price \$12.00.

The title of the book is deceptive, as the transcendent considered is

$$Z(\xi) = 2ie^{-\xi^2} \int_{-\infty}^{i\xi} e^{-t^2} dt, \quad \xi = x + iy,$$

which is essentially the error function of complex argument. The related function  $w(z) = i\pi^{1/2}Z(z)$  has been previously tabulated [1]. The present volume gives tables for  $Z$  and  $Z'$  mostly to 6S (see the remarks below) for the range  $x = 0(0.1)10$ ,  $y = -10(0.1)10$ . The authors seem unaware of other tables of the error function for complex argument [2, 3, 4, 5].

Some properties of  $Z(\xi)$  and figures which show its behavior are presented. The method of computation is described. We have noted two typographical errors.

On p. 3, line 3, read  $Y(x) \equiv x^{-1} \exp(-x^2) \int_0^x \exp(t^2) dt$ , and on p. 6, in the continued fraction representation for  $Z(\xi)$ , for  $\xi^2 + 5/2$ , read  $-\xi^2 + 5/2$ . Also on p. 6 the sign of  $a_{n+1}$  in the difference equations for  $A_n$  and  $B_n$  should be negative. If  $1 < |y| \leq 10$ ,  $0 \leq x \leq 10$ , a well-known continued fraction representation based on the asymptotic expansion of  $Z$  was employed. This representation converges in the entire complex plane except for points on the real axis. Near the real axis, the number of terms required for convergence increases, and maintenance of accuracy was difficult, owing to underflow and overflow. Accordingly, in the region  $|y| \leq 1$ ,  $0 \leq x \leq 10$ , the entries were found by numerical integration of the differential equation satisfied by  $Z(\xi)$ . For  $y = 0$ ,  $5 < x < 10$ , the entries for  $\text{Im}(Z)$  and  $\text{Im}(Z')$  become very small, and significance is gradually lost. At  $x = 9$ , the first four figures are significant, but thereafter the entries are not reliable. In this region, one should use  $\text{Im}(Z(x)) = \pi^{1/2}e^{-x^2}$ .

It should be noted that even though the continued fraction convergents do not converge on the real axis, they may still be used for computational purposes in the same way that one uses an asymptotic expansion. In fact, the second-order convergent  $-\xi(\xi^2 - 5/2)/(\xi^4 - 3\xi^2 + 3/4)$  approximates  $Z(\xi)$  to almost 6 decimal places if  $x \geq 0$ ,  $y \geq 0$  and  $|\xi| \geq 5$ . Thus, use of the above approximation, together with the formula connecting  $Z(\xi)$  and  $Z(\bar{\xi})$ , would have considerably reduced the bulk of the present tables.

Y. L. L.

1. V. N. FADDEEVA & N. M. TERENT'EV, *Tablicy značeniĭ funktsii*

$$w(z) = e^{-z^2} \left( 1 + 2i\pi^{-1/2} \int_0^z e^{t^2} dt \right)$$

*ot kompleksnogo argumenta*, Gosudarstv. Izdat. Tehn.-Teor. Lit., Moscow, 1954. Also available as *Tables of Values of the Function*  $w(z) = e^{-z^2} \left( 1 + 2i\pi^{-1/2} \int_0^z e^{t^2} dt \right)$ , Pergamon Press, 1961. See *Math. Comp.*, v. 16, 1962, p. 384-387.

2. P. C. CLEMMOW & C. M. MUNFORD, "A Table of  $(\pi/2)^{1/2}e^{1/2i\pi\rho^2} \int_{\rho}^{\infty} e^{-1/2i\pi\lambda^2} d\lambda$  for complex values of  $\rho$ ," *Philos. Trans. Roy. Soc. London*, ser. A, v. 243, 1952, p. 189-211. See *MTAC*, v. 7, 1953, p. 178.

3. K. A. KARPOV, *Tablicy funktsii  $w(z) = e^{-z^2} \int_0^z e^{z^2} dx$  v kompleksnoĭ oblasti*, Insdat. Akad. Nauk SSSR, Moscow, 1954. See *MTAC*, v. 12, 1958, p. 304-305.

4. K. A. KARPOV, *Tablitsy funktsii  $F(z) = \int_0^z e^{z^2} dx$  v kompleksnoĭ oblasti*, Izdat. Akad. Nauk SSSR, Moscow, 1958. See *Math. Comp.*, v. 14, 1960, p. 84.

5. R. HENSMAN & D. P. JENKINS, "Tables of  $(2/\pi)e^{z^2} \int_z^\infty e^{-t^2}$  for complex  $z$ ," UMT file, *Math. Comp.*, v. 14, 1960, p. 83.

11 [L].—FRITZ OBERHETTINGER & T. P. HIGGENS, *Tables of Lebedev, Mehler, and Generalized Mehler Transforms*, Math. Note No. 246, Boeing Scientific Research Laboratories, Seattle, 1961, 48 p., 21.5 cm.

The transform pairs tabulated are:

A. (Lebedev)

$$g(y) = \int_0^\infty f(x)K_{ix}(y) dx,$$

$$f(x) = 2\pi^{-2}x \sinh \pi x \int_0^\infty y^{-1}K_{ix}(y)g(y) dy$$

where  $K_\nu(x)$  is the modified Bessel function of the second kind.

B, C. (Mehler, Generalized Mehler)

$$g(y) = \int_0^\infty f(x)P_{ix-1/2}^k(y) dx$$

$$f(x) = \pi^{-1}x \sinh \pi x \Gamma(\tfrac{1}{2} - k + ix) \Gamma(\tfrac{1}{2} - k - ix) \int_1^\infty g(y)P_{ix-1/2}^k(y) dy,$$

where  $P_{ix-1/2}^k(y)$  is the Legendre function. The Mehler transform is the case  $k = 0$ . Furthermore,  $k = \frac{1}{2}$  and  $k = -\frac{1}{2}$  give rise to Fourier cosine and sine transforms, respectively.

Most of the results given here are new. A list of Lebedev transforms is available in *Tables of Integral Transforms* by A. Erdélyi, W. Magnus, F. Oberhettinger, and F. G. Tricomi, McGraw-Hill, 1954, v. 2, Ch. 12, but the present compilation is much more extensive. Only a few entries of the Mehler transform are given in the above reference.

The transforms are useful to solve certain boundary-value problems of the wave or heat conduction equation involving wedge or conically shaped boundaries, and a number of references to physical problems are given in the bibliography. To facilitate use of the tables, definitions of higher transcendental functions which enter into the transforms are provided in a separate section.

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12 [W].—F. P. FOWLER, JR., *Basic Mathematics for Administration*, John Wiley & Sons, Inc., New York, 1962, xvii + 339, 23.5 cm. Price \$7.95.

This book presents a general survey of basic mathematics used in the development of modern decision-making techniques. The authors give a background sketch