

bounds is in order. While the lower bound would be particularly important, the improved upper bound would also be useful.

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3. JOHN W. WRENCH, JR., "Evaluation of Artin's constant and the twin-prime constant," *Math. Comp.*, v. 15, 1961, p. 396-398.
4. PAUL T. BATEMAN & ROSEMARIE M. STEMMLER, "Waring's problem in algebraic number fields and primes of the form $(p^r - 1)/(p^d - 1)$," *Illinois J. Math.*, v. 6, 1962, p. 142-156.

An Approximation to the Fermi Integral $F_{1/2}(x)$

By H. Werner and G. Raymann

The Fermi Integral as defined, for instance, in the *Handbuch der Physik*, Bd. XX, S. 58 [1], is given by

$$(1) \quad F_p(x) = \int_0^\infty \frac{t^p}{e^{t-x} + 1} dt.$$

The function $F_{1/2}(x)$ has for negative values of x an expansion of the form

$$(2) \quad F_{1/2}(x) = \frac{\sqrt{\pi}}{2} \sum_{\nu=1}^{\infty} (-1)^{\nu-1} \cdot \frac{e^{\nu x}}{p^{3/2}},$$

and for large positive x the asymptotic expansion

$$(3) \quad F_{1/2}(x) \sim x^{3/2} \left[\frac{2}{3} + \frac{\pi^2}{12 \cdot x^2} + \left(\frac{1}{3} \right) \cdot \frac{7}{60} \cdot \frac{\pi^4}{x^4} + \dots \right. \\ \left. + \left(2n - 1 \right)^{\frac{1}{2}} \frac{2^{2n-1} - 1}{n} |B_{2n}| \cdot \frac{\pi^{2n}}{x^{2n}} + \dots \right];$$

compare [2], formulas (10) and (12);

B_{2n} are the Bernoulli numbers, given for example in [3], page 298. We obtained Chebyshev approximations to $F_{1/2}(x)$, based upon the table by McDougall and Stoner [4]. This table was subtabulated by interpolation with a fifth-degree polynomial. The approximations are

$$(4) \quad F_{1/2}^*(x) = e^x \sum_{\nu=0}^5 a_\nu e^{\nu x} \quad \text{for } -\infty < x \leq +1, \\ F_{1/2}^*(x) = x^{3/2} \left[\frac{2}{3} + \sum_{\nu=0}^5 \frac{b_\nu}{x^{2\nu+2}} \right] \quad \text{for } +1 < x < +\infty,$$

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the coefficients

ν	a_ν	b_ν
0	+0.8860 7596	+0.8435 00
1	-0.3087 1705	+0.7108 09
2	+0.1463 8520	-3.7124 56
3	-0.0584 3877	+6.7056 28
4	+0.0143 1771	-5.5948 77
5	-0.0015 0176	+1.7777 87

With these approximations, the relative error $|F_{1/2}(x) - F_{1/2}^*(x)|/F_{1/2}(x)$ is less than $2 \cdot 10^{-4}$ and $5 \cdot 10^{-4}$, respectively.

Another intensive table of $F_p(x)$ has been given by G. A. Chisnall [5] who also discusses in [6] a method for the interpolation of the existing tables of $F_{1/2}(x)$. It is not difficult to obtain analogous Chebyshev approximations to $F_p(x)$ for any fixed values of p to a prescribed degree of accuracy if one is able to generate the function with this (or slightly more) accuracy.

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On the Congruences $(p-1)! \equiv -1$ and $2^{p-1} \equiv 1 \pmod{p^2}$

By Erna H. Pearson

The results of computations to determine primes p such that one of the relations

$$(1) \quad (p-1)! \equiv -1 \pmod{p^2},$$

$$(2) \quad 2^{p-1} \equiv 1 \pmod{p^2}$$

holds have been published previously [1-5]. The known Wilson primes (those satisfying (1)) are 5, 13, and 563, the last having been determined by Goldberg [3] in testing $p < 10^4$. Froberg [4] tested $10^4 < p < 30,000$ without finding additional Wilson primes.

Froberg [4] determined $p = 1093$ and $p = 3511$ to be the only primes less than