the coefficients

ν	a_ν	b_ν
0	+0.8860 7596	+0.8435 00
1	−0.3087 1705	+0.7108 09
2	+0.1463 8520	−3.7124 56
3	−0.0584 3877	+6.7056 28
4	+0.0143 1771	−5.5948 77
5	−0.0015 0176	+1.7777 87

With these approximations, the relative error $|F_{1/2}(x) - F^*_{1/2}(x)|/F_{1/2}(x)$ is less than $2 \cdot 10^{-4}$ and $5 \cdot 10^{-4}$, respectively.

Another intensive table of $F_p(x)$ has been given by G. A. Chisnall [5] who also discusses in [6] a method for the interpolation of the existing tables of $F_{1/2}(x)$. It is not difficult to obtain analogous Chebyshev approximations to $F_p(x)$ for any fixed values of p to a prescribed degree of accuracy if one is able to generate the function with this (or slighty more) accuracy.

Institut für Angewandte Mathematik
der Universität Hamburg, Hamburg, and
Deutsches Rechenzentrum
Darmstadt, Germany

1. O. Madelung, Halbleiter in *Handbuch der Physik*, herausgegeben von S. Flügge, Bd. XX, Springer, Berlin, 1957, p. 58–59.
2. A. C. Beer, M. N. Chase & P. F. Choquard, "Extension of McDougall-Stoner tables of the Fermi-Dirac functions," Phys. Acta Helv. 28, 1955, p. 529–542.
3. E. Jahnke & F. Emde, unter Mitwirkung von F. Lösch, *Tafeln höherer Funktionen*, B. G. Teubner Verlagsgesellschaft, Stuttgart, 1960.
4. J. McDougall & E. C. Stoner, "The computation of Fermi-Dirac functions," *Trans. Roy. Soc., Ser. A* 237, 1939, p. 67–104.
5. G. A. Chisnall, *Jodrell Bank Annals*, v. 1, 1956, p. 126–140.
6. G. A. Chisnall, "A modified Chebyshev Everett interpolation formula," *MTAC* v. 10, 1956, p. 66–73.

On the Congruences $(p-1)! \equiv -1$ and $2^{p-1} \equiv 1 \pmod{p^2}$

By Erna H. Pearson

The results of computations to determine primes p such that one of the relations

$$(1) \qquad (p-1)! \equiv -1 \pmod{p^2},$$

$$(2) \qquad 2^{p-1} \equiv 1 \pmod{p^2}$$

holds have been published previously [1–5]. The known Wilson primes (those satisfying (1)) are 5, 13, and 563, the last having been determined by Goldberg [3] in testing $p < 10^4$. Froberg [4] tested $10^4 < p < 30{,}000$ without finding additional Wilson primes.

Froberg [4] determined $p = 1093$ and $p = 3511$ to be the only primes less than

Received April 6, 1962.

50,000 satisfying (2). Kravitz [5] extended the range of primes tested in (2) to $p < 10^5$ and found no additional primes of this type.

The author recently tested primes $30{,}000 < p \leqq 200{,}183$ in (1) and $10^5 < p \leqq 200{,}183$ in (2). No primes satisfying either relation were found in these ranges.

The computations were carried out on the Control Data 1604 Computer at The University of Texas. The formula used as a basis for programming the computations was

$$(p - 1)! \equiv (-1)^{(p-1)/2}\, 2^{2p-2}\, ([(p - 1)/2]!)^2 \pmod{p^2} \tag{3}$$

for an odd prime p. This formula is given as Theorem 133, Hardy and Wright [6].

The primes not exceeding 200,183 were generated and stored on tape, from which they were called in blocks to be tested individually. The 1604 computer is a binary computer with a 48-bit word length. The residue of 2^{p-1} was determined by successions of left shifts and reductions modulo p^2, where the left shifts were long enough to multiply each intermediate residue by a reasonably large power of 2, yet short enough to avoid end-around carry. This residue was tested in (2), then squared, and reduced for use as a factor in (3). The residue of $[(p - 1)/2]!$ was built up by successive multiplication and reduction modulo p^2, finally being squared and reduced for use in (3). The computation time per prime was roughly proportional to the size of the prime, about 8.5 seconds for a prime of the order of 10^5.

The author wishes to express her appreciation for the considerable amount of machine time given by The University of Texas Computation Center for these computations, as well as for the encouragement given her by the staff of the Center and by H. S. Vandiver.

The University of Texas
Austin, Texas

EDITORIAL NOTES: (1) The 18,000th prime is 200,183; (2) the residues were not saved and are not available for comparison.

1. N. G. W. H. BEEGER, "On the congruence $(p - 1)! \equiv -1 \pmod{p^2}$," *Mess. of Math.* v. 49, 1920, p. 177–178.

2. EMMA LEHMER, "A note on Wilson's quotient," *Am. Math. Month.*, v. 44, 1937, p. 237 and 462.

3. K. GOLDBERG, "Table of Wilson quotients and the third Wilson prime," *Proc. London Math. Soc.*, v. 28, 1953, p. 252–256.

4. C. E. FROBERG, "Some computations of Wilson and Fermat remainders," *MTAC*, v. 12, 1958, p. 281.

5. S. KRAVITZ, "The congruence $2^{p-1} \equiv \pmod{p^2}$ for $p < 100{,}000$," *Math. Comp.*, v. 14, 1960, p. 378.

6. G. H. HARDY & E. M. WRIGHT, *An Introduction to the Theory of Numbers*, Third Edition. London, Oxford, 1954, p. 106.