1. FRANCIS L. MIKSA, *Table of quadratic partitions* $x^2 + y^2 = N$, RMT **83**, *MTAC*, v. 9, 1955, p. 198.
2. JOHN LEECH, "Some solutions of Diophantine equations," *Proc. Cambridge Philos. Soc.*, v. 53, 1957, p. 778–780.

39[F].—DAVID C. MAPES, *Fast Method for Computing the Number of Primes less than a Given Limit*, Lawrence Radiation Laboratory Report UCRL-6920, May 1962, Livermore, California. Table of 20 pages deposited in UMT File.

This report is the original writeup of [1]. The table in [1] gives $\pi(x)$, $Li(x)$, $R(x)$, $L(x) - \pi(x)$ and $R(x) - \pi(x)$ for $x = 10^7(10^7)10^9$, where $\pi(x)$ is the number of primes $\leqq x$, and $Li(x)$ and $R(x)$ are Chebyshev's and Riemann's approximation formulas. The table here gives the same quantities for $x = 10^6(10^6)10^9$. It thus has greater "continuity," but not enough to trace the course of $\pi(x)$ unequivocally.

For example, Rosser and Schoenfeld [2] have recently proved that $\pi(x) < Li(x)$ for $x \leqq 10^8$. While it is highly probable that this inequality continues to $x = 10^9$, the gaps here, of $\Delta x = 10^6$, would appear to preclude a *rigorous* proof at this time. Study of the table, however, shows no value of x for which $\pi(x)$ approaches $Li(x)$ sufficiently close to arouse much suspicion. The relevant function is

$$PI(x) = \frac{Li(x) - \pi(x)}{\sqrt{x}} \log x,$$

and for $313 \leqq x \leqq 10^8$, Appel and Rosser [3] showed a minimum value of $PI(x)$, equal to 0.526, at $x = 30{,}909{,}673$. Here (and also in [1]) one finds values of 0.615 and 0.543 at $x = 110 \cdot 10^6$ and $180 \cdot 10^6$, respectively. It is thus likely that a value of $PI(x)$ less than 0.526 can be found in the neighborhood of these x (especially the second), but it is unlikely that $PI(x)$ becomes negative there. The relevant theory [4] is made difficult by incomplete knowledge of the zeta function. In the second half of the table, $x > 500 \cdot 10^6$, no close approaches at all are noted, and $Li(x) - \pi(x)$ exceeds 1000 there, except for $x = 501 \cdot 10^6$, $604 \cdot 10^6$, and $605 \cdot 10^6$.

The low values of $PI(x)$ are always associated with the condition $\pi(x) > R(x)$. The largest value of $R(x) - \pi(x)$ shown here is $+914$, for $x = 905 \cdot 10^6$.

D. S.

1. DAVID C. MAPES, "Fast method for computing the number of primes less than a given limit, "*Math. Comp.*, v. 17, 1963, p. 179–185.
2. J. BARKLEY ROSSER AND LOWELL SCHOENFELD, "Approximate formulas for some functions of prime numbers," *Illinois J. Math.*, v. 6, 1962, p. 64–94.
3. KENNETH I. APPEL AND J. BARKLEY ROSSER, *Table for Functions of Primes*, IDA-CRD Technical Report Number 4, 1961; reviewed in RMT **55**, *Math. Comp.*, v. 16, 1962, p. 500–501.
4. A. E. INGHAM, *The Distribution of Prime Numbers*, Cambridge Tract No. 30, Cambridge University Press, 1932.

40[F].—J. BARKLEY ROSSER & LOWELL SCHOENFELD, "Approximate formulas for some functions of prime numbers," *Illinois J. Math.*, v. 6, 1962, Tables I-IV on p. 90–93.

The four number-theoretic tables reviewed here were presented by the authors in connection with their proofs of numerous inequalities concerning the distribution of primes. These inequalities include

$$\frac{x}{\log x}\left(1 + \frac{1}{2 \log x}\right) < \pi(x) < \frac{x}{\log x}\left(1 + \frac{3}{2 \log x}\right) \qquad (59 \leqq x),$$

$$\frac{x}{\log x - \frac{1}{2}} < \pi(x) < \frac{x}{\log x - \frac{3}{2}} \qquad (67 \leqq x),$$

$$n\,(\log n + \log\log n - \tfrac{3}{2}) < p_n < n\,(\log n + \log\log n - \tfrac{1}{2}) \qquad (20 \leqq x),$$

and

$$li(x) - li(\sqrt{x}) < \pi(x) < li(x) \qquad (11 \leqq x \leqq 10^8).$$

Table II is an excerpt from a table computed several years earlier by Rosser and R. J. Walker on an IBM 650. It lists the four functions, $\theta(x) = \sum_{p \leqq x} \log p$, $\sum_{p \leqq x} p^{-1}$, $\sum_{p \leqq x} p^{-1} \log p$, and $\prod_{p \leqq x} p/(p - 1)$ to 10D for $x = 500(500)16{,}000$. Here the sums and the product are taken over the primes not exceeding x. For larger values of x see [1].

Table III lists $\psi(n) - \theta(n)$ to 15D for each of the 84 values of n equal to a prime power, $p^a (a > 1)$, which is less than $50{,}653 = 37^3$. The function $\psi(x) - \theta(x)$ remains constant between such prime-powers, and at such numbers the function increases by a jump equal to $\log p$.

Table I (which, for convenience, we describe out of order) is concerned with bounds on $\psi(x)$. The table lists 120 pairs of numbers, ϵ and b, such that

$$(1 - \epsilon)x < \psi(x) < (1 + \epsilon)x \quad \text{for} \quad e^b < x.$$

For example, $\epsilon = 4.0977 \cdot 10^{-6}$ for $b = 4900$.

Finally, Table IV lists $-\zeta'(n)$, $-\zeta'(n)/\zeta(n)$, and $\sum_p p^{-n} \log p$ to 17D for $n = 2(1)29$. Here ζ is the Riemann zeta function. This table was computed with the Euler-Maclaurin formula on an electronic computer. The values obtained were checked by a different computation and agree with Walther's 7D table of $-\zeta'(n)/\zeta(n)$ [2], and Gauss's 10D value of $-\zeta'(2)$ [3]. The purpose in computing Table IV was to use it in evaluating the limit:

$$\sum_{p \leqq x} p^{-1} \log p - \log x \to -1.33258227573322087.$$

However, Table IV certainly has other uses; for example, the reviewer has recently used it in [4].

D. S.

1. Kenneth I. Appeland J. Barkley Rosser, *Table for Estimating Functions of Primes*, IDA-CRD Technical Report Number 4, 1961; reviewed in RMT **55**, *Math. Comp.* v. 16, 1962, p. 500–501.
2. A. Walther, "Anschauliches zur Riemannschen Zetafunktion," *Acta. Math.*, v. 48, 1926, p. 393–400.
3. C. F. Gauss, *Recherches Arithmétiques*, Blanchard, Paris, 1953, p. 370.
4. Daniel Shanks, "The second-order term in the asymptotic expansion of $B(x)$," *Notices*, Amer. Math. Soc., v. 10, 1963, p. 261, Abstract 599–46. For errata see *ibid.*, p. 377.

41[F, G, X].—Gabor Szegö et al, Editors, *Studies in Mathematical Analysis and Related Topics—Essays in Honor of George Pólya*, Stanford University Press, Stanford, 1962, xxi + 447 p., 25 cm. Price $10.00.

This substantial volume, consisting primarily of sixty new research papers by leading mathematicians, was published on December 13, 1962, Professor Pólya's seventy-fifth birthday. The topics are of a great variety and include analysis, topology, algebra, number theory, and applied mathematics. While many of the papers begin with an opening paragraph that mentions some related work of Pólya, they have no other common theme.