useless for determining the normal rounding). In the remaining 415 comparisons, there are five discrepancies greater than one final unit at:

ν	0	1	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$
x	8.4	9.9	9.0	9.6	9.9

The greatest discrepancy is at $\nu = 1$, $x = 9.9$, where the B.A. value (for $E = -\Omega$) is $-0.251012\frac{1}{2}$, and the present tables have -0.25100, a difference of about $1\frac{1}{4}$ final units. The accuracy of the B.A. tables appears to be excellent, but a 6D table only partially investigated cannot provide quite the check that a good 10D table does. Nevertheless, this provides further reason to think that about one per cent of the Bernard and Ishimaru values differ from the true values by more than one final unit. There are also four cases in which the rounding is correct by the normal rule, instead of by the hypothetical rule. Of the five additional 6D values of $E_\nu(\nu)$ given in Brauer & Brauer [3], that for $\nu = 0.1$ ends in zero and so is useless for testing the hypothesis; those for $\nu = 0.2(0.1)0.5$ are all positive and all rounded upwards in Bernard & Ishimaru, so that they conform to the hypothesis.

The discussion given above is unavoidably partial and incomplete for $\nu \geqq -\frac{1}{2}$, while for $\nu < -\frac{1}{2}$ it merely shows that the hypothesis needs modifying, without discovering how it should be modified. One would welcome some statement by the authors on a subject which might on occasion be of great interest to users of the tables. Lacking information, users will presumably have either to delve into analytical details and the FORTRAN program, or to accept rounding uncertainties of a size which an authoritative statement might almost halve. The work involved in the discussion, tentative as it is, has been felt to be worthwhile, because those who are interested in special higher mathematical functions are likely to rank the tables of Bernard and Ishimaru among the most important produced in that field since automatic computers began to contribute.

A. F.

1. A. Fletcher, J. C. P. Miller, L. Rosenhead &. L. J. Comrie, *An Index of Mathematical Tables*, second ed., vol. 1, 1962, p. 458. Blackwell, Oxford, England (for scientific Computing Service, London); American ed., Addison-Wesley.
2. British Association for the Advancement of Science, *Mathematical Tables*, vol. 10, 1952, p. 180, Cambridge Univ. Press.
3. P. Brauer & E. Brauer, *Z. Angew. Math. Mech.*, vol. 21, 1941, p. 177–182, especially p. 180–181.
4. British Association for the Advancement of Science, *Reports* for 1923, p. 293; for 1924, p. 280; for 1925, p. 244. London.

50[L].—E. Paran & B. J. Kagle, *Tables of Legendre Polynomials of the First and Second Kind*, Research Report 62-129-103-R1, Westinghouse Research Laboratories, Pittsburgh, Pennsylvania, Sept. 7, 1962, i + 202 p., 28 cm.

These are tables of the functions $P_n(x)$ and $Q_n(x)$, in the usual notation. Since $P_n(x)$ and $Q_n(x) - P_n(x) \tanh^{-1} x$ are polynomials, $Q_n(x)$ is not, so that the word "functions" could profitably replace "polynomials" in the title. $P_n(x)$ is tabulated for $x = 0.001(0.001)1$ and $Q_n(x)$ for $x = 0.001(0.001)0.999$, in both cases for $n = 0(1)27$ and to 6S without differences. At $x = 1$, $Q_n(x)$ is infinite, and the numbers given in the tables are limiting values of $Q_n(x) - P_n(x) \tanh^{-1}x$, although this is not explained in the very brief accompanying text. It is a pity that the values of $P_n(0)$ and $Q_n(0)$ are not given. The tables were calculated on an IBM 7090 computer, and it is stated that the accuracy of the values is about ± 2 in the last significant figure.

The limit $n = 27$ is considerably higher than for previous tables of $P_n(x)$ and especially $Q_n(x)$, though $P_n(\cos \theta)$ has been tabulated for still higher values of n.

Most of the values of $P_n(x)$ up to $n = 16$ may be checked against a 6D table of Tallqvist [1]; this is insufficient near zeros, where Paran & Kagle keep 6S. Other checking tables also exist. A few random comparisons suggest good accuracy in the present table.

In 1945 the Admiralty Computing Service issued a useful little 5D table of $Q_n(x)$, $n = 0(1)7$, $x = 0(0.01)1$, mainly copied from Vandrey with corrections. The reviewer has compared all 792 common values with Paran & Kagle, and a few slight corrections to the ACS table are given in the appropriate section of this issue (p. 335). The table of Paran & Kagle has most often one extra figure, so that the comparison checks it only very partially indeed, but here again one has the impression of good accuracy in the Paran & Kagle values. If these values contain any errors as large as two final units, perhaps they occur for the higher values of n.

This is a "working table" rather than a definitive one, with properly rounded values, but it is important enough to make one glad that it has been classified as for unlimited circulation.

A. F.

1. H. Tallqvist, *Sechsstellige Tafeln der 16 ersten Kugelfunktionen $P_n(x)$*, *Acta. Soc. Sci. Fenn.*, Nova Ser. A, Tom II, No. 4, 43 p., Helsingfors, 1937.

51[L, M].—Yudell L. Luke, *Integrals of Bessel Functions*, McGraw-Hill Book Company, New York, 1962, xv + 419 p., 23 cm. Price $12.50.

This book professes to deal with definite and indefinite integrals involving Bessel functions (and related functions) and purports to provide the applied mathematician with the basic information relating to such integrals; but, in fact, it does more than it promises. In addition to information relating to Bessel functions, it gives much useful information about other special functions, and the reader learns a great deal about the evaluation, convergent expansion, and asymptotic expansion of integrals involving functions of the hypergeometric type. Within the field of integrals involving Bessel functions, special emphasis is placed on indefinite integrals, since these are somewhat scantily treated in other well-known and easily accessible works of reference where definite integrals are more adequately covered.

Chapter I is preparatory and contains information and useful collections of formulas regarding the gamma function, generalized hypergeometric series, and Bessel functions; in the latter case including polynomial approximations useful for the numerical computation of these functions. A brief list of tables of Bessel functions is appended.

Chapter II is devoted to the integral

$$(1) \qquad Wi_{\mu,\nu}(z) = \int_0^z t^\mu W_\nu(t)\,dt$$

in which W is J, Y, $H^{(1,\,2)}$, I, or K. Since the organization of this chapter is typical of the organization of several other chapters, it is worth considering it in some de-