Chapter XI lists a great many indefinite integrals whose integrands contain a product of two Bessel functions, or a product of a Bessel function and a Struve function. The integrals are evaluated by utilizing the differential equations satisfied by these functions. There is also a brief section on an integral over a product of three Bessel functions.

The last chapter on indefinite integrals, Chapter XII, contains a miscellany of integrals that do not fit into the classification of earlier chapters. Some samples are:

$$e^{-y} \int_0^x e^{-t} I_0(2(yt)^{1/2})\, dt; \qquad \int^z e^{-\alpha}(\alpha t^2 + \beta)^{-2} t^{\nu} J_{\nu-1}(S)\, dt,$$

where $2\alpha(\alpha t^2 + \beta) = \gamma(t^2 - 1)$, $S(\alpha t^2 + \beta) = \gamma t$, and α, β, γ are independent of t; and similar and related integrals.

Many definite integrals can be obtained from the indefinite integrals of the earlier chapters; others, which cannot be so obtained are collected in Chapter XIII. In view of existing collections, the author emphasizes the numerical and analytical results obtained since about 1945 and 1950, respectively, but the more important earlier results are also included. Some of the groups of definite integrals listed here are: integrals expressing orthogonal properties (in Fourier-Bessel and Neumann series); convolution integrals (including Sonine's and related integrals); Lommel's functions of two variables; Hankel's, Weber's, Weber-Schafheitlin's, Sonine-Gegenbauer's, and related infinite integrals; infinite integrals involving products of Bessel functions; integrals with respect to the order. In many of these cases references to numerical tables are given in addition to analytical results. There is also a brief section on dual and triple integral equations.

Chapter XIV (38 p.) contains a useful collection of numerical tables of Bessel functions (10 tables) and their integrals (2 tables); these are extracted from published works.

A bibliography of 18 pages, an index of notation, author index, and subject index complete the volume.

The volume is reproduced by photo offset from typed copy. Both the typing and the reproduction are excellent.

The value of such a compilation depends essentially on how practical the grouping of integrals will prove in actual use, how easy it is (for a non-expert) to find a given integral, and how well the author succeeded in keeping down the number of (inevitable) misprints. Meanwhile, the first impression is decidedly favourable, and there is every prospect of the book becoming a valuable work of reference.

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52[L, M].—H. C. SPICER, *Tables of the Inverse Probability Integral*

$$P = \frac{2}{\sqrt{\pi}} \int_0^\beta e^{-\beta^2}\, d\beta,$$

U. S. Geological Survey, Washington 25, D. C. Deposited in the UMT File.

This manuscript is in the form of original computation sheets. It contains inverse values for $P = \frac{2}{\sqrt{\pi}} \int_0^\beta e^{-\beta^2}\, d\beta$: with P ranging in value as indicated: [0(0.0001) 0.9] 9D; [0.9(0.00001)0.99997]9D.

On each sheet the column indicated by P, the argument, is followed immediately on the same line with the 9-decimal-place value for beta. The values following the beta values on the intervening lines, indicated as $\Delta_1 P$, are the differences between successive pairs of beta values. Only the last decimal place is tabulated in the argument except at the 0.0010's values, where all are given. The digits in the values of beta to the right of the decimal point are omitted when they continue downward the same in the tabulation, except at the 0.0010's values, where all are given.

AUTHOR'S SUMMARY

53[L, S].—JOSEPH HILSENRATH & GUY G. ZIEGLER, *Tables of Einstein Functions—Vibrational Contributions to the Thermodynamic Functions*, National Bureau of Standards, Monograph 49, U. S. Government Printing Office, Washington, D. C., 1962, vii + 258 p., 26 cm. Price $2.75.

It is a well-known theorem of statistical mechanics that a harmonic oscillator (or any degree of freedom of a complex molecule quantized in the same way) contributes the following to the (Gibbs) free energy, enthalpy, entropy, and heat capacity:

$$-(F^\circ - E_0^\circ)/RT = -\ln(1 - e^{-x})$$

$$(\mathrm{H}^\circ - E_0^\circ)/RT = xe^{-x}(1 - e^{-x})^{-1}$$

$$S^\circ/R = xe^{-x}(1 - e^{-x})^{-1} - \ln(1 - e^{-x})$$

$$C^\circ{}_p/R = x^2e^{-x}(1 - e^{-x})^{-2}$$

where $x = hc\nu/kT$. This book contains first a table of the above dimensionless quantities for $x = 0.0010\ (.001)\ 0.1500\ (.001)\ 4.00\ (.01)\ 10.00\ (.2)\ 16.0$. A second table gives $-(F^\circ - E_0^\circ)/T$, S°, and $C^\circ{}_p$ (all in calories/mole-deg) for $T = 273.15°$K, 298.15°K, 400°K and thence by 100° intervals to 5000°K. For each temperature the frequency ν (in cm^{-1}) runs from 100 to 4000 in steps of 10. All results are stated to 5 decimal places, and the accuracy is claimed to be better than one-half a unit in the last place.

These tables would be far more useful if harmonic oscillators were more common components of molecules and crystals. Unfortunately, most vibratory degrees of freedom are not very harmonic, and accurate computations of thermodynamic properties require corrections for anharmonicity. These corrections cannot easily be applied to the final thermodynamic functions, but rather tend to require a complete new calculation.

In this reviewer's opinion the need for tables like these is passing. With computers now very generally available, the essential content of these tables could have been stored much more conveniently in the form of a set of subroutines in standard machine languages like FORTRAN. This would make the results available where they are most needed: as inputs to machine calculations for specific molecules and crystals.

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