primes $p = 1531$ and 1543; listing of the prime powers as $p = 9, 25$, etc.; and the continued listing of the arguments in some power-residue tables after the table has ended. However, these are demerits in aesthetics, and while they should have been corrected, they do not nullify the high utility of the tables.

D. S.

1. L. McKee, C. Nichol & J. Selfridge, *Indices and Power Residues for all Primes and Powers Less than 2000*; reviewed in RMT **64**, *Math. Comp.*, v. 15, 1961, p. 300.
2. J. C. P. Miller, *Table of Least Primitive Roots*; one copy deposited in UMT File. (See *Math. Comp.*, v. 17, 1963, p. 88–89, RMT **2**.)
3. K. G. J. Jacobi, *Canon Arithmeticus, sive tabulae quibus exhibentur pro singulis numeris primis vel primorum potestatibus infra 1000 numeri ad datos indices et indices ad datos numeros pertinentes*, Berlin, 1839.

73[F].—Daniel Shanks, *Solved and Unsolved Problems in Number Theory*, Vol. 1, Spartan Books, Washington, D.C., 1962, ix + 229 p., 24 cm. Price $7.50.

This book is an excellent introduction to number theory, well motivated by an entertaining and instructive account of the origin and history of the classical problems connected with perfect numbers, primes, quadratic residues, Fermat's Last Theorem, and other topics.

Superb in every respect, as an introductory account, as a history of number theory, as an essay in mathematical and scientific philosophy, this volume can be used either as a textbook in high school or college, as a book for self-study, or as a gift to the educated layman with the perennial query, "What does a mathematician do?"

This delightful and stimulating book should be on the shelf of anyone interested in mathematics.

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74[G].—I. M. Gel'fand, *Lectures on Linear Algebra*, Interscience Publishers, Inc., New York. 1961 ix + 185 p., 23 cm. Price $6.00.

The author presents in this book a very clearly written shorter text on linear algebra which would generally be suitable for a one-semester course at the junior level in the United States. The contents consist of four chapters (Chapter 1, n-Dimensional Spaces; Chapter 2, Linear Transformations; Chapter 3, The Canonical Form of an Arbitrary Linear Transformation; and Chapter 4, Introduction to Tensors), the first two chapters comprising about three-fourths of the book. The author is to be congratulated for his lucid discussions and proofs. The notation and the printing are excellent.

For those who wish to use this as a text, it should be mentioned that the author frequently assumes knowledge of results from matrix theory that American students, as opposed to Russian students, do not possess at this level.

R. S. V.

75[I, L].—F. W. J. Olver, *Tables for Bessel Functions of Moderate or Large Orders* (National Physical Laboratory, *Mathematical Tables*, v. 6), Her Majesty's Stationery Office, London, 1962, iii + 51 p., 28 cm. Price 17s. 6d. (In U.S.A.:

British Information Services, 845 Third Avenue, New York 22, New York. Price $3.50.)

Various tables directly tabulating Bessel functions of fairly high order exist, and are briefly described in this volume, but such direct tabulation cannot be extended indefinitely, and needs to be supplemented by a method of computing any Bessel function of high order. After describing J. C. P. Miller's algorithm, the author sets out his own asymptotic expansions, which form the basis of the present work. The slim volume packs in a small compass a great deal of information, divided almost equally between text and tables, and it would exceed the proper limits of a review to describe either in full detail. The author's aim is to give tables to facilitate the computation of $J_n(nx)$, $Y_n(nx)$, $I_n(nx)$, $K_n(nx)$, and their first derivatives to ten significant figures (except in the immediate neighborhood of zeros) when $n \geqq 10$.

In the case of J, Y, J', and Y', the asymptotic expressions used involve an auxiliary variable ζ along with Airy functions Ai, Bi and their derivatives Ai', Bi' of argument $n^{2/3}\zeta$; here

$$\frac{2}{3}\zeta^{3/2} = ln\,\frac{1 + (1 - x^2)^{1/2}}{x} - (1 - x^2)^{1/2} \qquad (0 < x \leqq 1)$$

$$\tfrac{2}{3}(-\zeta)^{3/2} = (x^2 - 1)^{1/2} - \sec^{-1} x \qquad (x \geqq 1)$$

ζ is tabulated against x, and various coefficients are tabulated against ζ. The British Association tables of Airy integrals are assumed to be available, and indeed it may be noted that the whole work grew out of the prolonged British Association and Royal Society labors on the tabulation of Bessel and related functions.

In the case of I, K, I', and K', the asymptotic expressions used involve an auxiliary variable ξ and exponential functions of $\pm\, n\xi$, where

$$\xi = (1 + x^2)^{1/2} - ln\,\frac{1 + (1 + x^2)^{1/2}}{x},$$

but it is found convenient to take the argument of all the tables to be $t = (1 + x^2)^{-1/2}$, so that

$$\xi = \frac{1}{t} - \frac{1}{2}\,ln\,\frac{1 + t}{1 - t}.$$

Either $\xi - x$ or $\xi - ln\, x$ is tabulated against t, as are various coefficients. The author forms exponentials in his worked examples by using the well-known National Bureau of Standards tables, but any other logarithmic or antilogarithmic tables (either common or natural) with sufficient figures could be pressed into service.

An interesting feature is that provision for interpolation is made by tabulating coefficients in "economized" polynomials, as described by Clenshaw & Olver [1]. The tables of $\zeta(x)$ give coefficients c_i for use with the formula

$$f_p = f_0 + c_1 p + c_2 p^2 + \cdots + c_n p^n \qquad (0 \leqq p \leqq 1)$$

where n never needs to exceed 5. The remaining tables give coefficients d_2, d_4 (no more than these two ever being needed) which are derived from even central differences and allow interpolation by the formula (akin to a modified Everett formula)

$$f_p = qf_0 + q(1 - q^2)\, d_{2,0} + q^3(1 - q^2)\, d_{4,0} + pf_1 + p(1 - p^2)\, d_{2,1} + p^3(1 - p^2)\, d_{4,1}$$

where $q = 1 - p$. The author believes that the present tables are the first to use these particular aids to interpolation, and states that he welcomes comments and criticisms by users.

A useful bibliography has some forty references. The whole work constitutes a powerful tool, not to be overlooked by anyone concerned with numerical values of Bessel functions of high order.

A. F.

1. C. W. CLENSHAW & F. W. J. OLVER, "The use of economized polynomials in mathematical tables," *Proc. Camb. Phil. Soc.*, v. 51, 1955, p. 614–628.

76[K].—DONALD MAINLAND, LEE HERRERA & MARION I. SUTCLIFFE, *Tables for Use with Binomial Samples*, Department of Medical Statistics, New York University College of Medicine, New York 16, N. Y., 1956, xix + 83 p.

These tables are a consolidation of tables previously published (Mainland [1], Mainland and Murray [2], Mainland and Sutcliffe [3]). They contain many more entries than the original versions, and sections have been recalculated to give finer precision. All the tables are for use with qualitative data, that is, of the A, *not* A type.

Tables I–IV are for the comparison of two binomial samples arranged in a 2×2 contingency table.

Sample	A	not A	
1 2	a b	c d	N_1 N_2
	a + b	c + d	$N_1 + N_2$

The labels A and *not* A are assigned arbitrarily.

Tables I and II give minimum contrast pairs a, b, $a < b$, which are significant at the two-tailed 5% and 1% levels, respectively. Such pairs a, b are tabulated for $N_1 = N_2 = N = 4(1)20(10)100(50)200(100)500$. For $N \leqq 30$, some pairs a, b were omitted because they can be obtained quickly on sight by interpolation. The portion up to $N = 20$ was based on the exact hypergeometric distribution, whereas, for $N \leqq 30$, the chi-square with Yates' correction was generally used, and the significance was tested by Table VIII of Fisher and Yates [4].

Table III contains single-tail exact probabilities to 4D of 2×2 contingency tables for equal samples up to $N = 20$. Pairs a, b and corresponding exact probabilities are tabulated for all pairs a, b such that the tail probabilities are less than or equal to one-half. These pairs are those for which $a + b \leqq N$ and $a < b$.

Table IV gives minimum contrasts and probabilities for unequal samples of size up to $N = 20$. For given sample sizes N_1 and N_2, $N_1 > N_2$, the table gives (i) the pairs a, b $\left(a \leqq \frac{N_1}{2}\right)$ which provide a minimum contrast for significance at the