

where  $q = 1 - p$ . The author believes that the present tables are the first to use these particular aids to interpolation, and states that he welcomes comments and criticisms by users.

A useful bibliography has some forty references. The whole work constitutes a powerful tool, not to be overlooked by anyone concerned with numerical values of Bessel functions of high order.

A. F.

1. C. W. CLENSHAW & F. W. J. OLVER, "The use of economized polynomials in mathematical tables," *Proc. Camb. Phil. Soc.*, v. 51, 1955, p. 614-628.

76[K].—DONALD MAINLAND, LEE HERRERA & MARION I. SUTCLIFFE, *Tables for Use with Binomial Samples*, Department of Medical Statistics, New York University College of Medicine, New York 16, N. Y., 1956, xix + 83 p.

These tables are a consolidation of tables previously published (Mainland [1], Mainland and Murray [2], Mainland and Sutcliffe [3]). They contain many more entries than the original versions, and sections have been recalculated to give finer precision. All the tables are for use with qualitative data, that is, of the  $A$ , not  $A$  type.

Tables I-IV are for the comparison of two binomial samples arranged in a  $2 \times 2$  contingency table.

Sample	A	not A	
1	a	c	$N_1$
2	b	d	$N_2$
	$a + b$	$c + d$	$N_1 + N_2$

The labels  $A$  and *not A* are assigned arbitrarily.

Tables I and II give minimum contrast pairs  $a, b$ ,  $a < b$ , which are significant at the two-tailed 5% and 1% levels, respectively. Such pairs  $a, b$  are tabulated for  $N_1 = N_2 = N = 4(1)20(10)100(50)200(100)500$ . For  $N \leq 30$ , some pairs  $a, b$  were omitted because they can be obtained quickly on sight by interpolation. The portion up to  $N = 20$  was based on the exact hypergeometric distribution, whereas, for  $N \leq 30$ , the chi-square with Yates' correction was generally used, and the significance was tested by Table VIII of Fisher and Yates [4].

Table III contains single-tail exact probabilities to 4D of  $2 \times 2$  contingency tables for equal samples up to  $N = 20$ . Pairs  $a, b$  and corresponding exact probabilities are tabulated for all pairs  $a, b$  such that the tail probabilities are less than or equal to one-half. These pairs are those for which  $a + b \leq N$  and  $a < b$ .

Table IV gives minimum contrasts and probabilities for unequal samples of size up to  $N = 20$ . For given sample sizes  $N_1$  and  $N_2$ ,  $N_1 > N_2$ , the table gives (i) the pairs  $a, b$  ( $a \leq \frac{N_1}{2}$ ) which provide a minimum contrast for significance at the

single-tail 2.5 % and 0.5 % levels, and (ii) 4D of the corresponding exact probabilities.

Tables V-IX provide upper and lower confidence limits for a single binomial ratio such as (number of  $A$ 's)/ $N$ , where  $N$  is the number of individuals in a random sample all of which have been classified as  $A$  or *not*  $A$ . Tables V-VIII were prepared from Table VIII<sub>1</sub> of Fisher and Yates [4], whereas Table IX was calculated directly from binomial expansions.

Table V gives 95 % and 99 % confidence limits for samples in which the number of  $A$ 's is 1 through 14,  $A$  being the label of the class having the fewer number of individuals. The range of  $N$  is approximately  $N = 2A(1)2A + 20$ , and thereafter by increasingly large intervals to  $N = 1000$ . The limits are given to at least two decimal places and at least two significant figures.

Table VI provides 95 % and 99 % confidence limits for samples in which the number of  $A$ 's is greater than 14. The tabulation is according to values of  $N$  and the percentage of  $A$ 's in the sample,  $P = 100$  (number of  $A$ 's)/ $N$ . The range of  $P$  (in per cent) is  $P = 0.1(.1)1(1)50$ . The range of  $N$  is not the same for all values of  $P$ ; but in all cases  $N$  extends, by increasingly large intervals, to  $N = 100,000$ . As  $P$  increases, the range of  $N$  increases downward and the number of values of  $N$  for which limits are given increases; i.e., for  $P = 0.1$  %,  $N$  has 14 values over the range 3,000 to 100,000, whereas for  $P = 50$  % there are 55 values of  $N$  over the larger range, 30 to 100,000. The limits are given to at least two decimal places and, in all but a few cases, to at least two significant figures.

Table VII has the same description as Table V except that the confidence limits given are 80 % limits.

Table VII is the same type as Table VI. This table contains 80 % confidence limits, but for fewer values of  $P$  than Table VI. The values of  $P$  are  $P = 0.1, 0.5, 1.0, 5(5)50$ . The values of  $N$  are approximately equal to those in Table VI.

Table IX gives *upper* 95, 99, and 80 % confidence limits for samples in which the number of  $A$ 's is zero, the lower limit being zero per cent  $A$ 's. The limits are given to 2D for  $N = 1(1)30(5)100(10)200, 220, 250, 300, 400, 500, 700, \text{ and } 1,000$ .

Table X is entitled "Percentages of Successful Experiments (% S) in Relation to Sample Size ( $N$ ) and to Percentages of  $A$ 's in Populations  $V$  and  $W$ ". Specifically, the table answers the following: If a sample of size  $N$  is taken from each of two effectively infinite populations which have  $P_1$  and  $P_2$  per cent  $A$ 's, and if the sample percentages of  $A$ 's are tested by means of Table I for a significant difference at the 5 per cent level, then, what percentage of such experiments would show the sample percentages to be significantly different and in the same direction as  $P_1$  and  $P_2$ . Values  $S$  in per cent are given to 1D for all pairs of  $P_1$  and  $P_2$ ,  $P_1 < P_2$ , obtained from the following set: 1, 5, 10, 15, 25, 33, 50, 67, 75, 85, 90, 95, and 99. The sample sizes are  $N = 5, 10, 15, 20, 30, 50, 70, \text{ and } 100$ .

The introduction to the ten tables contains fourteen examples which demonstrate the use of the tables. Several of these examples show how to interpolate or extrapolate in certain of the tables. Included also in the introduction is a section on the preparation and reliability of the tables.

For use in testing  $2 \times 2$  contingency tables with equal samples, Tables I and II are certainly more convenient for the user than other existing tables. The test is

done easily and quickly by eye. If, instead of "assigning the label  $A$  arbitrarily", we pick  $a$  to be the smallest of all four entries, a large number of entries in Tables I and II can be eliminated, making the test seem still easier.

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1. D. MAINLAND, "Statistical methods in medical research. I. Qualitative statistics (enumeration data)", *Canad. J. Res.*, Series E, v. 26, 1948, p. 1-166.

2. D. MAINLAND & I. M. MURRAY, "Tables for use in fourfold contingency tests," *Science*, v. 116, 1952; p. 591-594.

3. D. MAINLAND & M. I. SUTCLIFFE, "Statistical methods in medical research. II. Sample sizes required in experiments involving all-or-none responses," *Canad. J. Med. Sci.*, v. 31, 1953, p. 406-416.

4. R. A. FISHER & F. YATES, *Statistical Tables for Biological, Agricultural, and Medical Research*, Hafner Publishing Company, Inc., New York, 1953.

77[K].—A. M. YAGLOM, *An Introduction to the Theory of Stationary Random Functions*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1962, xiii + 235 p., 23 cm. Price \$10.60.

This book is an exceedingly well written account of the subject of stationary processes and linear prediction theory. It does not require much mathematical background on the reader's part. The small quantity of Hilbert space theory needed for prediction theory is supplied by the author. Bochner's fundamental theorem (also due to Khinchin) on the representation of positive definite functions as Fourier transforms of finite nonnegative measures is not proved but is discussed at length. The theorem due to Szëgo, and Kolmogoroff's generalization, that

$$\inf_p \frac{1}{2\pi} \int_{|z|=1} |1 - zp(z)|^2 f(\theta) d\theta = \exp \left\{ \frac{1}{2\pi} \int \log f(\theta) d\theta \right\},$$

where  $p$  runs through all polynomials in  $z$ ,  $f(\theta) \geq 0$  in  $L_1$ , and its corollaries characterizing deterministic discrete and continuous processes, are also examined but not proved. The problem of linear extrapolation is therefore restricted to cases in which the optimal estimate can be expressed as an infinite series of past values:

$$\hat{\xi}_m = a_1 \xi_{-1} + a_2 \xi_{-2} + \cdots + a_k \xi_{-k} + \cdots,$$

$m \geq 0$ , in which the series converges in  $L_2(f(\theta))$ ,  $f$  being the power density. The case in which  $f(\theta)$  is a rational function of  $e^{i\theta}$  is treated in detail. The extrapolation problem is followed by a chapter on linear filtering; that is, given  $\zeta_n$  for  $n \leq -1$ , with  $\zeta_n = \xi_n + \eta_n$ , find the optimal estimate of  $\xi_m$ . The problem is solved in detail for the case in which  $\xi$  and  $\zeta$  have power densities rational in  $e^{i\theta}$ . The problems of extrapolation and filtering for random functions (instead of sequences) come next, with a chapter for each. This new edition concludes with two short appendices: one on generalized random processes, in which "white noise," for example, can be defined rigorously; and the other, written by D. B. Lowdenslager, on some recent developments, in particular, vector-valued processes.

Although the reviewer does not read Russian, it must be concluded on the basis of the beautiful style of the book that the translator, R. A. Silverman, has done an excellent job of translating or writing, or both. This book will provide both for