beginners and for more experienced mathematicians a fundamental grasp of the applied aspects of linear prediction theory, and is highly recommended.

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78[K].—A. H. ZALUDOVA, "The non-central t-test (q-test) based on range in place of standard deviation," *Acta Technica*, v. 5, 1960, p. 143–185.

This paper presents tables of percentage points of the non-central q-distribution and additional tables which were used to compute these percentage points. The non-central q is a non-central t-type statistic, which is based on the mean sample range. Because of the ease with which it is used, the non-central q-test is proposed as a substitute for the non-central t-test as given by Johnson and Welch [1], especially when the application is in sampling inspection by variables.

The statistic q is given by

$$q(m, n) = \frac{T - \bar{x}}{\bar{w}(m, n)},$$

where $\bar{w}(m, n)$ is the mean of the m ranges of m sub-samples, each of size n, drawn from a normal population $N(\mu, \sigma)$; $\bar{x}$ is the over-all sample mean; and

$$T = \mu + K_p\sigma,$$

where K_p is the normal deviate exceeded with probability p. The distribution of $q(m, n)$ is derived by the author.

Tables 1, 2, 4, and 5 give percentage points q_ϵ to 3D for percentiles $\epsilon = .05, .95, .25$, and $.75$. For each value of ϵ the points are tabulated for $m = 1(1)5, n = 3(1)12$, and $p = .20, .10, .05, .02, .01$, and $.001$. Some details of the computation of the values and remarks on their accuracy are given.

In order to calculate the percentage points q_ϵ it was necessary to tabulate the frequency functions $f_m(\bar{w}_m)$ of the sample range and mean range, $\bar{w}_m$ denoting the mean range in m independent sub-samples from a normal population having unit standard deviation. This was done only for combinations $m = 1, 2, 4$ and $n = 3, 4, 6, 8, 10, 12$. These values of $f_m(\bar{w}_m)$ are given in Tables 7, 8, and 9. All values are given to at least 5D. They are tabulated for the following intervals of $\bar{w}_m$: $\bar{w}_1$ (single range) $= 0.0(0.1)8.2$, $\bar{w}_2 = 0.0(0.1)6.5$, and $\bar{w}_4 = 0.1(0.1)5.5$. These three tables were used to calculate a framework table of values of q_ϵ from which values of q_ϵ for other combinations of m and n were obtained by interpolation. Remarks on the calculation and accuracy of the values of $f_m(\bar{w}_m)$ are made.

It was found desirable for interpolation to know the limiting value of q_ϵ for $m = \infty$. The limiting distribution of q is derived and found to be concentrated at the point $E(q) = K_p/d_n$, where d_n is the expected value of the range in samples of size n from a normal population with unit standard deviation. Table 3 gives values of K_p/d_n to 5D for $n = 3(1)12$ and values of K_p corresponding to $p = .20, .10, .05, .02, .01$, and $.001$.

One additional table (Table 6) is presented; it shows a comparison of the non-central q-distribution with the power function of Lord's [2], [3] central t-test (u-test). This is done by tabulating $q_{.05}$ against an approximation to it which is based on the power function of Lord's u-test. The comparison is made for combinations $m = 1, 2, 4$ and $n = 3, 4, 6, 8, 10$. For each of the fifteen combinations a value of K_p is chosen such that α(error of first kind) $= \beta$(error of second kind) $= .05$, approximately. Values of $q_{.05}$ corresponding to these K_p are compared with the approximated values as a rough check on the tabulated values of q_α.

In conjunction with the tables of percentage points, twelve figures are given which show the relation (almost linear) between q_ϵ and K_p for p ranging from .001 to .20 (extended to .50 in some figures). For each value of ϵ and m, the relationship is shown for various values of n from 3 to 12. These graphs make possible the determination of q_ϵ for other than the six tabulated values of p.

Several examples are given which demonstrate the application of the non-central q-test in industrial sampling inspection. These illustrations indicate that the non-central q-test and corresponding tables of percentage points can be very useful in such work.

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1. N. L. Johnson & B. L. Welch, "Applications of the non-central t distribution," *Biometrika*, v. 31, 1940, p. 362–389.
2. E. Lord, "The use of range in place of standard deviation in the t-test," *Biometrika*, v. 34, 1947, p. 41–67.
3. E. Lord, "Power of the modified t-test (u-test) based on range," *Biometrika*, v. 37, 1950, p. 64–77.

79[L].—S. L. Belousov, *Tables of Normalized Associated Legendre Polynomials*, Pergamon Press, Ltd., Oxford, distributed by The Macmillan Company, New York, 1962, 379 p., 26 cm. Price $20.00.

This is a republication in an attractive binding of Russian tables of normalized associated Legendre polynomials previously reviewed in this journal (*MTAC*, v. 11, 1957, p. 276, RMT **115**).

For convenience the contents are here summarized again. The polynomials $\bar{P}_n^m(\cos\theta)$ considered are related to the associated Legendre polynomials P_n^m by the normalizing factor $\left[\frac{2n+1}{2}\frac{(n-m)!}{(n+m)!}\right]^{1/2}$, so that $\int_{-1}^{1}[\bar{P}_n^m(x)]^2\,dx = 1$, and are herein tabulated to 6D for $m = 0(1)36$, $n = m(1)56$, and $\theta = 0(2°.5)90°$. No tabular differences are given.

The introduction has been translated into English by D. E. Brown.

These useful tables remain the most extensive of their kind published to date.

J. W. W.

80[L].—O. S. Berlyand, R. I. Gavrilova, & A. P. Prudnikov, *Tables of Integral Error Functions and Hermite Polynomials*, Pergamon Press, Ltd., Oxford, England, distributed by The Macmillan Co., New York, 1962, 163 p., 26 cm. Price $15.00.

This volume of the Pergamon Mathematical Tables Series is an English translation by Prasenjit Basu of original tables of integral error functions and Hermite polynomials published in Minsk in 1961 by the Byelorussian Academy of Sciences.