

“Scientific Applications for the UNIVAC LARC” by Cecil E. Leith, Jr.

“Digital Computers in Communications Engineering” by Robert M. Fano.

“Automatic Programming for Numerically Controlled Tools—APT III” by Edgar A. Bates.

“Medical Diagnosis Aided by Digital Computers” by Robert S. Ledley.

“CLASS—The Automated Classroom (Philco 2000)” by Donald E. Englund and D. P. Estavan.

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**87[X].**—WOLFGANG HAHN, *Theory and Applications of Liapunov's Direct Method*, 1963, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, x + 182 p., 23.5 cm. Price \$9.00.

Take a differential equation

$$(1) \quad \dot{x} = X(x, t)$$

where  $x$  and  $X$  are  $n$ -vectors (vectors with  $n$  components),  $X(0, t) = 0$  for  $t \geq 0$ , and  $X$  satisfies standard conditions, continuity included, to guarantee existence and uniqueness of solutions in a certain spherical region  $S: \|x\| < A$ . How stable are the solutions relative to the “trivial” solution  $x = 0$ ? The problem of stability thus faced is of great theoretical and practical importance. It has given rise to a widely developed theory whose creator is the great Russian mathematician Liapunov (around 1890). Owing in part to the language barrier (in spite of a French translation in 1907), this theory lay practically forgotten by the world at large. It woke up very suddenly about 40 years ago and has been vigorously pursued in the USSR since then. In recent years it has at last reached these shores, with the result that many younger scientists in the U.S. are now participating in its development.

The present volume, a translation of the 1959 German edition of the *Ergebnisse* series, is a most valuable and timely addition to the literature in English on the subject. It is part careful treatise, part summary of many contributions, for a large part from the Soviet Union, the leader in this field. Definitions and theorems are carefully stated and proved, making this monograph a very good guide in the subject, particularly in view of its ample bibliography. The following chapter titles will give an idea of the extensive ground covered:

1. Fundamental concepts.
2. Sufficient conditions for stability or instability of equilibrium.
3. Application of the stability theorems to concrete problems.
4. The converse of the main theorems.
5. Liapunov functions with certain properties of rate of change.
6. The sensitivity of the stability behavior to perturbations.
7. Critical points.
8. Generalizations of the concept of stability.

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