

88[X].—WILLIAM KARUSH, *The Crescent Dictionary of Mathematics*, The Macmillan Co., New York, 1962, x + 313 p., 24 cm. Price \$6.50.

From the introduction: "The subject matter of the 1422 entries in this dictionary covers two general categories. First, a detailed treatment is provided of the following standard high school and college mathematics material: arithmetic; elementary, intermediate, and college algebra; plane and solid analytic geometry; differential and integral calculus. Second, a wide selection of items is provided from more advanced mathematics, including the following fields: logic and fundamental concepts; theory of equations, theory of numbers, and modern higher algebra; advanced calculus; geometry and topology; probability and statistics; recent areas such as computer sciences, information theory, operations research, and so on."

The coverage in the first "general category" is thorough, but most branches of advanced mathematics are hardly attempted. For instance, even advanced calculus, including such terms as "Jacobian", "Stieltjes Integral", "Uniform Convergence" and "Cauchy's Theorem", is not covered. On the other hand some modern specialties, e.g., Theory of Games, Linear Programming, Computers and Their Coding, Logic, Machine Intelligence, Markoff Process, etc. are covered briefly but well. This class of specialties is apparently favored because of the author's position on the staff of the System Development Corporation.

The coverage is not as broad as that of the larger volume [1] of James and James. Perhaps because of this narrower range, the present volume is more uniform in character. The definitions are uniformly good and are carefully written and printed. With its limited extent in mind, the dictionary can be recommended to the several audiences for which it was intended.

D. S.

1. GLENN JAMES & ROBERT C. JAMES, *Mathematics Dictionary*, 2nd ed., D. Van Nostrand Co., Princeton, New Jersey, 1959. (Reviewed in RMT 66, *MTAC*, v. 13, 1959, p. 331-332.)

89[X].—N. N. KRASOVSKII, *Stability of Motion*, Stanford University Press, Stanford, California, 1963, vi + 188 p., 24 cm. Price \$6.00.

One of the most powerful and flexible techniques available for the study of the stability of solutions of functional equations is that based upon the use of Liapunov functions.

To illustrate the basic idea, first presented by Liapunov in his fundamental memoir of 1892, consider the system of equations

$$(1) \quad \frac{dx_i}{dt} = g_i(x_1, x_2, \dots, x_N), \quad i = 1, 2, \dots, N,$$

or, in vector notation, $dx/dt = g(x)$. Considering $V(x)$, where V is as yet undetermined, as a function of t , we have

$$(2) \quad \frac{dV}{dt} = \sum_i \frac{\partial V}{\partial x_i} g_i = (\text{grad } V, g).$$

Suppose that V , as a function of x , has been chosen so that

$$(3) \quad (\text{grad } V, g) \leq -kg, \quad k > 0,$$

for all x . Then (2) and (3) yield the result that $V \leq V(0)e^{-kt}$, whence $V(x) \rightarrow 0$ as $t \rightarrow \infty$. If $V(x)$ is a function such as $\sum_i x_i^2$, with the property that $V(x) \rightarrow 0$ if and only if $x \rightarrow 0$, we have deduced in this way the important fact that $x(t) \rightarrow 0$ as $t \rightarrow \infty$, a stability result.

The problem, of course, lies in obtaining $V(x)$, given $g(x)$. Although there is no uniform approach, there exists a vast literature of results due to mathematicians such as Cetaev, Malkin, Persidskii, Massera, Letov, and others. An excellent survey may be found in another recent book in this area, namely, J. P. LaSalle and S. Lefschetz, *Stability by Liapunov's Direct Method with Applications*, Academic Press Inc., New York, 1962.

The great merit of Krasovskii's book is to contain not only a more complete and detailed account of the research of this nature in the field of ordinary differential equations, but also to present a thorough discussion of the application of these methods to differential-difference and more general time-lag equations.

The book is wholeheartedly recommended to all those interested in the modern theory of differential equations and in modern control theory.

The format is attractive, the price is reasonable, and the translation by J. L. Brenner is excellent.

RICHARD BELLMAN

90[X].—JAMES B. SCARBOROUGH, *Numerical Mathematical Analysis*, Fifth Edition, Johns Hopkins Press, Baltimore, Md., 1962, xxi + 594 p., 23.5 cm. Price \$7.00.

This is a revised edition of the well-known text by James B. Scarborough. In addition to a number of corrections and minor changes, the Fifth Edition contains a chapter on Newton's interpolation formula for unequal intervals. It is gratifying that the author has been able to find time periodically to review and improve one of the oldest and most popular elementary texts in the field of Numerical Analysis.

H. P.

91[X, Z].—GEORGE S. SEBESTYEN, *Decision-Making Processes in Pattern Recognition*, The Macmillan Company, New York, 1962, viii + 162 p., 24 cm. Price \$7.50.

Pattern recognition is a subject which is currently receiving considerable attention. It is important in a variety of situations ranging from the need of the Post Office for mechanical reading devices to speed up sorting of the mails to the need of the Military to be able to decide whether an incoming radar or sonar signal comes from a harmless object such as a meteor or a fishing boat, or whether it comes from a threatening source such as a missile warhead or a hostile submarine. In any situation, the problem to be solved is how to organize one's knowledge about the object in question and how to be able to compare this with similarly organized knowledge about the possible categories to which the object can be assigned.

In the book under review, the author attempts to exploit a geometrical point of view. Data describing a given object consist of numerical values assigned to N