

On the Factors of Certain Mersenne Numbers, II

By John Brillhart

This paper is the second of two papers dealing with the prime factors of Mersenne numbers $M_p = 2^p - 1$ of prime exponent (see Brillhart and Johnson [1]). The 899 new factors given below, which were discovered on the IBM 7090 of the Computing Facility at the University of California at Los Angeles, constitute the remaining factors needed for a complete listing in the literature of all prime factors $q < 2^{35}$ for $103 \leq p \leq 257$ and $q < 2^{34}$ for $257 < p < 20000$ (See Brillhart [2], Karst [4], Kravitz [5], Riesel [6], [7]).

In pursuing the present investigation, all factors in the literature were rediscovered, which has thus permitted a thorough checking of the tables cited above. Complete agreement was obtained with the tables of Karst and Kravitz, while errors were found in Riesel [7]. (See [9]. Also see Selfridge [8]). It should be noted that the three small factors omitted in Riesel [7] for $p = 1451$ and 1459 are given in the table below. Also, it should be mentioned that the factors marked in the following table with an asterisk were discovered earlier by E. Karst.

The method of search was essentially the same as in [1]. The complete set of factors produced by the program was tested for correctness and multiplicity by a special program written for that purpose. All factors were found to be correct, and none was found to be multiple, thus adding further weight to the conjecture that no multiple factor of M_p exists for p a prime.

The factor limit used by the search program was the first multiple of $23040p = 4p \phi(3 \cdot 5 \cdot 7 \cdot 11 \cdot 13)$ greater than 2^{34} (here ϕ is the Euler function). Of the trial divisors $> 2^{34}$ only 18504622999, which is a divisor of M_{19963} , turned out to be a factor.

The running time for each M_p , which is an inverse linear function of p , varied from less than one minute for $p \approx 20000$ to approximately fifteen minutes for $p = 263$.

A primary concern in the present investigation was the discovery of all M_p , $3300 < p < 5000$, which had factors $< 2^{34}$. These M_p , being composite, were of interest in a search for *prime* Mersenne numbers over the same interval, since they could automatically be excluded from that search. (See Hurwitz [3]). No doubt the following table will be of considerable value to investigators in search of large Mersenne primes, who proceed beyond the present range.

The author would like to thank John Selfridge and the other directors of the UCLA Computing Facility for their very generous support of this project.

University of San Francisco
San Francisco, California

1. J. BRILLHART & G. D. JOHNSON, "On the factors of certain Mersenne numbers," *Math. Comp.*, v. 14, 1960, p. 365-369.

2. J. BRILLHART, "Some miscellaneous factorizations," *Math. Comp.*, v. 17, 1963, p. 447-450.

3. A. HURWITZ, "New Mersenne primes," *Math. Comp.*, v. 16, 1962, p. 249-251.

Received April 15, 1963.

4. E. KARST, "New factors of Mersenne numbers," *Math. Comp.*, v. 15, 1961, p. 51.
 5. S. KRAVITZ, "Divisors of Mersenne numbers $10,000 < p < 15,000$," *Math. Comp.*, v. 15, 1961, p. 292-293.
 6. H. RIESEL, "Mersenne numbers," *MTAC*, v. 12, 1958, p. 207-213.
 7. H. RIESEL, "All factors $q < 10^8$ in all Mersenne numbers $2^p - 1$, p prime $< 10^4$," *Math. Comp.*, v. 16, 1962, p. 478-482.
 8. J. SELFRIDGE, Table Errata, *MTAC*, v. 13, 1959, p. 142.
 9. JOHN D. BRILLHART, *MTE 340*, *Math. Comp.*, v. 17, 1963, p. 486.

TABLE OF FACTORS

p	factors	p	factors	p	factors
383	7435494593	2081	1279128271	4271	1595303921
389	4765678679	2087	561653441		7988521111
431	4642152737		7784142689	4273	719086079
479	6293443049	2089	16673027617	4297	7216656809
541	4312790327	2347	205803737	4327	946574521
641	1173835097	2383	1132387303	4337	159211271
661	1330270433	2389	403411319	4447	155573849
677	1943118631	2437	3205142401	4451	196315807
743	1219280833	2473	4268907439	4463	1498603993
	14904366017	2503	192030161	4519	6537366161
769	1591805393	2531	193550633	4547	205815409
797	2006858753	2633	179654857	4597	116579921
827	1637241673	2677	502012457	4637	222056657
853	2065711807	2707	3822652153	4639	1557006127
907	3256645177	2833	356346073	4649	273314711
937	2419437071	2837	138871151	4751	268982617
991	8218291649	2861	7715659241		3274778783
1063	1485761479	2879	2770265929	4789	6854888399
1091	551978359	2917	114457247	4933	3637821119
1129	833798113	2957	10347489241	4957	8179079743
1217	1045741327	3001	5339517247	4969	879473249
1249	2379005273	3011	3105262367	4999	1094271103
1289	15856636079	3121	6805328017	5059	179331433
1303	1140690503	3169	507743519	5147	4185169817
1327	6363071161	3181	5621991247	5167	131696497
1361	137450113	3187	14452484089	5179	998770151
1373	1957392737	3229	137374577	5209	1945613591
	4540035943	3347	119588311	5323	2372950817
1433	16532942303	3373	2042978879	5399	5272458239
1447	1754380423	3407	138685343	5417	200515673
1451	174121	3449	565822247	5431	6040879577
	696481	3463	3052267423	5531	14639904343
1459	93377		17154129799	5623	1324014073
1481	3915278233	3529	487249031	5651	2828212481
1493	3788526319		1417154647		3346929073
1511	3828330041	3613	206916511	5657	218122607
1523	273887183	3617	8384488127	5659	867830287
	1117808897	3643	1627896409	5683	2436643081
1553	14194270913	3677	827972153	5743	13432968889
1579	132272831	3727	205342793	5801	200204113
1601	158098751	3733	2563787071	5843	3069690167
1613	165545417	3767	2467754167	5881	662671081
1627	6128193121		8943340177	5903	238433977
1667	137507497	3797	16689030041	5927	198969391
1721	1487078239	3803	3816820103	5939	706741001
1723	381913289		5905762367	5981	3883379567
1783	107022793	3823	8756696191	6029	249359441
1871	112667879	3833	14193959303	6067	677222809
	1500590647	3929	160020313	6101	807711391
1901	516125303	3947	1277249201	6277	1163404289
1931	543012649	4019	829167929	6301	150959359
1933	645881023	4057	9421717153	6311	1958694583
1951	16866820319	4079	102823433	6373	207849023
1973	4651860481	4099	1816586623		590432959
1993	162389641	4133	11173615097	6379	4972928063
2027	5405684681	4201	3612691961	6427	3959918927
2063	8271280799	4217	1398424673	6491	12296978807

TABLE OF FACTORS—Continued

<i>p</i>	factors	<i>p</i>	factors	<i>p</i>	factors
6553	10286532433	8803	226906129	11047	13433153
6563	179209279		7563748873		355116863
6619	471762607	8867	9284174617		2911370569
6653	4305755071	8933	1478072047	11071	3971411263
6689	142168007	8969	1142273903	11087	6883896127
6737	1106754361	9029	455621399	11119	978473
6793	13261715767	9103	2454405479		106319879
6827	108958921	9161	12187849367	11149	7672987079
6829	8286786631	9221	168025063	11171	501913031
6863	1608865639		5660569039	11287	13554264839
6869	257051719	9239	800633263	11317	16568089
6961	584208887	9293	4598548121	11321	29230823
6967	448730537	9349	1793044711	11329	445614887
6977	389414279	9371	7281660583	11437	309142111
6997	3572164417	9439	273523343	11471	144075461
7019	990366863	9467	1340299993	11491	7646938753
7027	373119647		4104001303	11497	21315439
7039	1057032553	9547	631801367	11503	58918367
	8541573097	9613	146559799	11527	20587223
7219	164362193		2253979337	11587	76659593
7283	102267887	9677	607618831	11617	43494049
7393	6525239233	9733	2932747561	11621	107378041
7411	133709263	9739	263751599	11633	67657529
	153496633	9743	564723767	11677	6137781511
7417	16269026327	9791	536076883	11699	561553
7457	7785883529	9923	2182682927	11701	187217*
	16694955311		8488689889		337339831
7529	209170679	10007	136255313	11719	267872903
7541	9711963409	10093	306665713	11777	373660657
7549	7460767289	10103	2804693831	11783	25451281
7583	5889276287	10139	4075879*	11833	1135969
7639	1840097599	10141	136599271		5522981753
7669	131093887	10193	69720121*	11903	942527153
7717	13968927551	10211	735193*	11909	18673313
7853	255002617	10223	1846703167	11969	12711079
7873	1200160121	10243	18847121*	11971	11611871
7877	517471639	10247	246419857		23175887
7879	259250617	10253	1293682529	11981	13011367
7907	546405329	10331	268607.*	12007	18658879
7951	498813937		4049753*	12037	2022217
7963	152889601	10337	163076513	12049	3819436609
7993	2066542193	10391	19389607*	12073	845111*
8017	585930463	10457	313711*		26729623
8039	968554799	10459	11632186031	12101	413951009
	11908411871	10499	14028847793	12107	8707160689
8053	219927431	10513	508471759	12113	30694343
8081	3498814409	10529	60962911	12119	8570677991
8087	360437591	10559	193546471	12157	40239671
8171	1089700903	10567	464292847	12163	12015608767
8231	14329002199	10601	3980103047	12211	4231258033
8243	4207243687	10627	10755119113	12277	39752927
8293	6874946759	10651	73981847		59715329
8353	154146263	10691	10200069281	12323	41208113
	297834569		11345203673		62107921
8419	4188149417	10733	310226633	12373	2282571041
8467	5910592559		465812201	12391	54049543
8501	221842097	10739	3810390503	12421	53559353
8537	147843767	10799	1024263553	12433	9546231463
8539	10089972727	10837	4115610839	12437	596977
8573	12254091887	10847	71590201	12451	15289230353
8627	123348847	10883	7574569	12457	11634839
8629	1337175913	10889	7584754729		302754929
8677	118753423	10903	40668191	12473	340662577
8689	1008827657	10949	1472136847	12479	9508999
	1639075583	10973	153623.*		125289161
		10979	18434641	12491	26006263
			270610393	12503	712996079

TABLE OF FACTORS—Continued

<i>p</i>	factors	<i>p</i>	factors	<i>p</i>	factors
12541	414429887	13759	10044071	15193	11759383
12583	805313·	13763	908359·	15217	119514319
	4454383		5613652441	15227	40564729
12589	14267541727	13807	5786568929	15241	243857·
12641	4045121	13831	1385866201		609641·
12647	850485457	13873	15620999		28043441
12653	2421353999	13877	23091329	15277	117571793
12659	2440047569	13901	14206823	15287	14339207
12671	99847481	13903	6812471	15307	4285961
12689	4344586711	13931	19085471	15319	153191·
12697	489164623	13997	291781463		164495423
12757	1837009	14071	7171031873	15331	7972121
12781	766246513	14087	22229287·	15391	15883513
12823	215554631·		2003875751	15401	123209
	15152990393	14107	11765239	15413	369913
12829	76871369	14149	424471	15427	7651793
12841	24038353	14177	74259127	15461	92767·
12889	203826647	14197	21806593·		1113193
12893	40097231·		90945983·	15467	1608569·
	246385231		130811159		3340873
12899	28480993	14281	13938257·	15473	7643663
12911	13659839		1805661079	15511	102372601·
12919	6006094777	14303	715151		286177951
12923	261251369	14323	332207663	15541	180151273·
12953	10723296487	14407	13068647329		348242729·
12959	570507017·	14419	439058551		1465081153
	636649753	14461	15849257	15583	249329
13003	1170271·	14519	409551953	15601	93607·
	13341079	14549	653075513		13073639·
13049	182687*	14591	12956809·		459418249
13063	554733359		1376952671	15641	594359
13103	20991007·	14593	933953·	15661	93967
	338398079		3012228689·	15679	752593
13109	3146161·		4211423057	15731	409007·
	76137073	14657	610317481·		7525207009
13147	121714927·		820440233	15737	193470679·
	1299028777	14669	352057		500531023
13151	11783297	14759	435360983	15739	168627647
13171	290631287	14767	57561767·	15749	28726177·
13183	698725367		373073489		29576623
13187	1967289409	14771	85819511	15761	504353
13259	3317189657	14779	10471571777	15767	882953·
13297	624107993	14783	4523599·		54396151
13331	9729097111		7480199	15791	31583
13367	2807071·	14813	37328761	15797	1033123801·
	192270929	14831	38708911		5782302287
13417	2395417513	14879	119033*	15803	31607
13451	107609·*	14891	251240953	15809	1305791783
	2690201	14923	11102713	15859	158591
13463	5497131383	14929	123970417	15877	1238407
13469	91023503·	14939	8276207·	15887	381289·
	1284053647		1560707209		762577·
13537	5929207·	15053	4696537		192550441
	10288121·	15061	451831	15889	970976791
	11993783	15073	90439	15901	477031·
13553	910219481	15091	271639·		87487303
13577	207429407		2661690217	15907	6012847
13669	956831·	15107	2084767	15913	95479
	1640281	15121	12429463·	15919	1782929
13679	246223·*		679326047	15923	31847
	100157639·	15131	272359	15971	119239487·
	312975521	15137	7034708833		194079593
13693	266493167	15161	11006887·	15973	479191
13709	212736263·		107915999	15991	159911
	1121944561	15173	2777963879	16001	288146009
13751	7528204967	15187	151871	16007	607753777

TABLE OF FACTORS—*Continued*

<i>p</i>	factors	<i>p</i>	factors	<i>p</i>	factors
16063	780533297	16871	303679	17609	3380929
16073	54519617		4082783	17623	5674607
16087	21138319	16883	33767		3736357969
	474663023		12054463		5384355191
16091	32183	16903	6795007	17627	384867919
	675823	16921	3643971193	17657	105943
16103	30144817	16927	304687		423769
16127	249774977	16931	33863		55513609
16139	34214681		3657097		18278491087
	128111383		537999457	17659	176591
16141	15850463	16937	135497	17681	106087
	119281991	16963	271409	17713	1133633
16183	161831		6683423	17737	2944343
16193	97159	16979	5331407	17747	1739207
16229	5193281	16981	4347137		5111137
	5529512423	16987	2649870079		33115903
16231	259697	17011	537683689	17749	48525767
	564319409	17027	442703	17783	569057
	9849392807		96032281		5548297
16249	3379793	17029	418879343		52104191
16267	780817	17033	102199	17791	1039883951
16273	390553		6234079	17837	354135799
	136790839		21359383	17839	3175343
16319	130553	17099	17269991		1810159009
	2074340729		231588857	17851	2142121
16339	653561	17107	23676089		3102111079
	32710679	17123	15843158489	17863	1249266769
16361	9816601	17159	34319	17891	9768487
16427	427103	17183	34367	17909	429817
16433	3449352433		721687	17921	1433681
16447	394729	17189	3437801		33655639
	2763097	17191	1547191		34945951
16451	1974121		75777929	17923	188585807
16477	185168527		9030088481	17939	35879
16481	98887	17207	59880361		143513
	63814433	17209	5758544417	17959	754279
16529	132233	17257	12942751	17977	22435297
16547	1048053887	17291	34583	17981	25712831
16561	99367		142339513	17987	7736748311
	1159271	17317	103903	18013	108079
16573	364607		774554777	18041	108247
16603	166031	17321	2632793	18043	4330321
	32541881		4676671	18049	49670849
16607	126777839		8452649		64507127
16651	139368871	17327	6228709961	18059	1625311
	3879849511	17333	249595201		1004983351
16657	499711	17351	34703	18061	541831
	7962047		4615367	18077	108463
16661	28923497	17377	799343	18089	191888113
16693	2270249	17383	4345751	18121	81182081
16699	300583	17389	2608351	18127	290033
16703	83782249	17401	206828287	18131	36263
16741	100447	17417	79421521	18143	471719
	1975439	17431	169603631		6966913
16747	167471	17443	418633		76672319
	7938079	17489	4861943	18191	36383
	6997164553	17497	419929	18199	172417327
16763	53004607		75866993	18211	3059449
16787	302167	17573	259799233	18217	109303
	1611553	17579	35159	18223	728921
16811	33623	17581	281297		219878719
16823	33647	17597	668687	18251	8906489
	2422513		7848263		15933999049
16829	235607	17599	11543633	18257	2336897
16831	302959		316783	18287	323935919
			7884353		6493823423

TABLE OF FACTORS—*Continued*

<i>p</i>	factors	<i>p</i>	factors	<i>p</i>	factors
18311	1025417	19073	160213201	19471	5607649
18313	30765841		440052257		1548956993
18329	1250441039	19081	114487	19489	14499817
18367	440809		7881330727	19541	156329
18379	1321266311	19121	917809	19559	39119
18427	184271	19139	13359023		996922231
18443	36887	19141	4555559	19571	6353725151
	85317319	19163	38327	19597	5865303713
18481	50231359		443240191	19603	4587103
18493	35025743	19183	1116757529		28110703
18503	7253177	19207	345727		12283553449
	4160510569		846222007		17171757529
18517	296273	19211	153689	19609	80004721
	121286351	19213	115279		133772599
18593	2677393	19237	461689	19661	1690847
18617	37680809		577111	19699	196991
18661	268681079	19249	1789348543		108108113
	4386380017	19259	1377288127	19717	3312457
18679	7807823		9325554463	19727	355087
18691	1233607	19267	462409		512903
18713	598817	19273	201981041		108459047
18731	37463	19289	154313	19739	2052857
	2526437281		118318727	19751	39503
18743	449833	19309	30701311	19763	48419351
	6185191	19319	38639	19777	909743
18749	44210143	19333	463993		26896721
18757	112543		2974498049	19793	23553671
18773	16073104687	19379	6201281	19819	3171041
18787	338167	19381	793380617		8561809
18797	150377	19391	38783	19841	91070191
	1616543		2909929807		2089773167
18803	37607	19403	9313441	19861	2539013089
18869	264167	19421	12235231	19867	171440153
18899	37799		212698793	19867	4648879
18917	100335769	19427	4446179783	19889	159113
	7907003329	19429	40062599	19913	6810247
18919	35265017	19447	311153	19919	39839
18973	4401737		1645216201		956113
	5919577	19457	155657		46052729
18979	3694717847		1147496033	19949	323643913
19001	632163271		5305340191	19963	1006785937
19037	723407	19469	1207079	19973	7660417
19069	6063943		4854478337	19991	18504622999
	35659031		8271171023	19997	26364361
					39983
					159977