7090 errors, four BESK errors (2957, 2969, 3049 and 3109), and an incorrect SWAC result for 1889. The SWAC (October 1962) confirmed the 7090 residue.

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1. ALEXANDER HURWITZ, "New Mersenne primes," *Math. Comp.*, v. 16, 1962, p. 249–251.
2. A. HURWITZ & J. L. SELFRIDGE, "Fermat numbers and perfect numbers," *Notices Amer. Math. Soc.*, v. 8, 1961, p. 601, abstract 587-104.
3. G. A. PAXSON, "The compositeness of the thirteenth Fermat number," *Math. Comp.*, v. 15, 1961, p. 420.
4. HANS RIESEL, "Mersenne numbers," *MTAC*, v. 12, 1958, p. 207–213.
5. RAPHAEL M. ROBINSON, "Mersenne and Fermat numbers," *Proc. Amer. Math. Soc.*, v. 5, 1954, p. 842–846.

Lucas' Test for Mersenne Numbers, $6000 < p < 7000$

By Sidney Kravitz and Murray Berg

Alexander Hurwitz [1] reported that he had applied Lucas' test to investigate the primality of the Mersenne Numbers $M_p = 2^p - 1$, p a prime, $3300 < p < 5000$, and discovered that M_{4253} and M_{4423} are prime numbers. Hurwitz [2] further states† that he tested all prime exponents between 5000 and 6000, where the corresponding M_p was not known to have a factor, without discovering any new Mersenne Primes.

TABLE

p	R	p	R	p	R
6007	07707	6247	00472	6659	75241*
6037	21420	6257	36710	6661	27165
6043	21605	6269	57356	6679	13275
6047	37000	6299	71037*	6701	07636
6053	53471	6329	25136*	6709	05700
6073	41646	6337	21676*	6733	35544
6079	15712	6359	51351	6763	01753
6089	32615	6361	10027*	6779	74306*
6091	02043	6451	23476	6791	41143
6133	42630	6469	51252	6823	14573*
6151	63451	6547	06546	6833	26431
6211	71252	6571	67142	6857	63102
6217	07377	6577	45051*	6907	46461*
6221	24166	6581	74210*	6911	63345
6229	06517	6599	77554	6971	65345
				6991	50365

The authors have tested the Mersenne Numbers $6000 < p < 7000$ without finding any new primes. A list of the five least significant octal digits of the S_{p-1}th remainder from the Lucas test ($S_1 = 4$) is given in the Table. Where a prime is missing from the list it indicates that a factor of the corresponding Mersenne Number was found by Riesel [3, 4] or that an unpublished† factor was found by

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† See pages 146, 87, and 93 of this issue of *Mathematics of Computation*.

John Brillhart. At the time of completion of these results we learned of similar work† by Donald B. Gillies on Illiac II. We compared our residues with his and found ten discrepancies. A check revealed that one of our three supposedly identical program decks contained an error. The questionable residues were recalculated and found to agree with Dr. Gillies' values. These residues are marked by an asterisk (*).

The authors have verified that Riesel's M_{3217} and Hurwitz's M_{4253} and M_{4423} are prime. Hurwitz's octal remainder [1] of 72013 for the prime exponent 3301 was also verified. The running time for p near 6500 was three hours, using an IBM 7090.

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1. A. Hurwitz, "New Mersenne primes", *Math. Comp.*, v. 16, 1962, p. 249–51.
2. A. Hurwitz, Private communication to the authors dated March 12, 1962.
3. H. Riesel, "Mersenne numbers", *MTAC*, v. 12, 1958, p. 207.
4. H. Riesel, "All factors $q < 10^8$ in all Mersenne numbers, $2^p - 1$, p a prime $< 10^4$", *Math. Comp.*, v. 16, 1962, p. 478–82. Errata, *Math. Comp.*, v. 17, 1963, p. 486.
5. S. Kravitz & M. Berg, "Recent research in Mersenne numbers" *Recreational Mathematics Magazine*, October, 1962. p. 40.

Note on the Congruence $a^{p-1} \equiv 1 \pmod{p^2}$.

By Hans Riesel

During 1961 the author with the aid of the electronic computer BESK gathered some data concerning the residues

$$a^{p-1} \pmod{p^2},$$

for odd primes p, from which the following tables have been compiled. In the case of $p < 1000$, the residues of $a^{p-1} \pmod{p^2}$ are available for comparison. For larger p the program has printed out only those residues $\equiv 0 \pmod{p^2}$. The running time on BESK was put at the author's disposal by courtesy of the Swedish Board for Computing Machinery.

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All odd primes $p < 500000$ with $a^{p-1} \equiv 1 \pmod{p^2}$ for $a \leq 10$.

a	p	a	p
2	1093, 3511	6	66161
3	11	7	5, 491531
4	1093, 3511	8	3, 1093, 3511
5	20771, 40487	9	11
		10	3, 487

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