14 [L].—F. M. ARSCOTT & I. M. KHABAZA, *Tables of Lamé Polynomials*, The Macmillan Company, New York, 1962, xxviii + 526 p., 27 cm. Price \$20.00.

Consider the Lamé equation

$$\frac{d^2w}{dz^2} + \{h - n(n + 1)k^2 sn^2 z\}w = 0 \tag{1}$$

where $sn\, z = sn(z, k)$ is the Jacobian sine-amplitude (elliptic) function. If $sn\, z = t$, we get

$$(1 - t^2)(1 - k^2t^2)\frac{d^2w}{dt^2} - t(1 + k^2 - 2k^2t^2)\frac{dw}{dt} + \{h - n(n + 1)k^2t^2\}w = 0 \tag{2}$$

Equation (1) may be transformed into four other forms like (2) by use of the substitutions $sn^2\, z = \zeta$, $k\, sn\, z = u$, $cn\, z = x$, and $dn\, z = y$. A trigonometric form results from putting $am\, z = v$ in (1). If in (1) we put $sn^2\, z = \zeta$ and then $\zeta = P(\xi)$, where $P(\xi)$ is the Weierstrass elliptic function, we then get the "Weierstrassian" form. Each form possesses finite solutions only when n is a positive integer and h is one of $(2n + 1)$ eigenvalues. For each form, these solutions fall into eight types. For example, for (1), they are of the form

$$w = sn^\rho\, z\, cn^\sigma\, z\, dn^\tau\, z\, F(sn^2\, z) \tag{3}$$

where $\rho, \sigma, \tau = 0$ or 1 and $F(sn^2\, z)$ is a polynomial in $sn^2\, z$ of degree $\frac{1}{2}(n - \rho - \sigma - \tau)$.

Let $N = n/2$ or $(n - 1)/2$, according as n is even or odd. For $N = 1(1)15$ and $C = k^2 = 0.1(0.1)0.9$, this volume gives to 6S the $(2n + 1)$ values of h and the corresponding coefficients of the polynomial F for each type. Similar tables for $N = 16(1)30$ are deposited with the Royal Society, in the Depository of Unpublished Mathematical Tables, as Reference 78. There are a few other scattered tables in the literature, but this appears to be the first systematic tabulation attempted.

An introduction clearly presents the basic properties of the functions, correspondence with other notations, and the method of computation. The computations were done on the Ferranti MERCURY computer, and the computer program is given. The tables were printed by photo offset from the computer output. The entries are legible, but the type is not pleasing to the eye.

Y. L. L.

15 [L].—G. BLANCH & DONALD S. CLEMM, *Tables Relating to the Radial Mathieu Functions, Vol. 1: Functions of the First Kind*, U.S. Government Printing Office, Washington 25, D.C., 1962, xxiv + 383 p., 27 cm. Price \$3.50.

This table provides numerical solutions of the differential equation

$$\frac{d^2f}{dz^2} - (a(q) - 2q \cosh 2z)f = 0 \tag{1}$$

where the $a(q)$ are the eigenvalues corresponding to which

$$\frac{d^2f}{dz^2} + (a(q) - 2q \cos 2z)f = 0 \tag{2}$$

has solutions of period π or 2π. The tabulated solutions depend on three parameters; namely q, z, and the order of the eigenvalue r.

The solutions of (2) fall into four categories, namely even or odd, and periodicity π or 2π. Solutions of (1) can be obtained from (2) by replacing z by iz. The even solutions of (1) are denoted by $Mc_r^{(1)}(z, q)$ and the odd ones by $Ms_r^{(1)}(z, q)$. For convenience, these are represented by

$$Mc_r^{(1)}(z, q) = M_r \cosh rz\, Pc_r(z, q)$$

$$Ms_r^{(1)}(z, q) = M_r \sinh rz\, Ps_r(z, q)$$

and

$$\frac{d}{dz} Mc_r^{(1)}(z, q) = rM_r \sinh rz\, Qc_r(z, q)$$

$$\frac{d}{dz} Ms_r^{(1)}(z, q) = rM_r \cosh rz\, Qs_s(z, q)$$

where

$$M_r = q^{1/2r}/(r!)2^{r-1}$$

Actually, the functions tabulated are the P and Q functions. The extraction of the hyperbolic functions leads to data which are readily interpolable in both z and q. The table must, therefore, be used in conjunction with a table of hyperbolic functions.

There are four basic tables. They provide 7D approximations to $Pc_r(x, q)$, $Qc_r(x, q)$, $Ps_r(x, q)$, and $Qs_r(x, q)$ for $q = 0\ (0.05)\ 1$; $r = 0\ (1)\ 7$, $x = 0\ (0.02)\ 1$, and $r = 8\ (1)\ 15$, $x = 0\ (0.01)\ 1$.

In addition, the values of $M_r(q)$ are furnished to 8S, as are those of the functions $C_r(q)$ and $S_r(q)$, which are defined on pages 1 and 197. The latter can be used instead of $M_r(q)$, corresponding to a different normalization. Also, the eigenvalues $a_r(q)$ and $b_r(q)$ are given to 8D. The computations were performed by a stepwise numerical integration of the differential equations for the P and Q functions. Some of the computations were performed on an 1103 ERA computer; the rest, on an IBM 7090.

The superscript appearing in $Mc_r^{(1)}(z, q)$ indicates that these are functions of the first kind (corresponding to Bessel functions for $q = 0$). A table for functions of the second kind is now in preparation.

Preceding the table is a good general discussion. A helpful chart relates the many non-standardized notations in this field.

Harry Hochstadt

Polytechnic Institute of Brooklyn
Brooklyn, New York

16 [L].—L. S. Bark & P. I. Kuznetsov, *Tablitsy tsilindricheskikh funktsiĭ ot dvukh mnimykh peremennykh* (*Tables of Cylinder Functions of Two Imaginary Variables*), Computing Center, Acad. Sci. USSR, Moscow, 1962, xx + 265 p., 27 cm. Price 2.87 rubles.

On replacing x and y in the Lommel functions of two variables

$$\sum_{m=0}^{\infty} (-1)^m \left(\frac{y}{x}\right)^{n+2m} J_{n+2m}(x)$$