has solutions of period π or 2π. The tabulated solutions depend on three parameters; namely q, z, and the order of the eigenvalue r.

The solutions of (2) fall into four categories, namely even or odd, and periodicity π or 2π. Solutions of (1) can be obtained from (2) by replacing z by iz. The even solutions of (1) are denoted by $Mc_r^{(1)}(z, q)$ and the odd ones by $Ms_r^{(1)}(z, q)$. For convenience, these are represented by

$$Mc_r^{(1)}(z, q) = M_r \cosh rz\, Pc_r(z, q)$$

$$Ms_r^{(1)}(z, q) = M_r \sinh rz\, Ps_r(z, q)$$

and

$$\frac{d}{dz} Mc_r^{(1)}(z, q) = rM_r \sinh rz\, Qc_r(z, q)$$

$$\frac{d}{dz} Ms_r^{(1)}(z, q) = rM_r \cosh rz\, Qs_s(z, q)$$

where

$$M_r = q^{1/2r}/(r!)2^{r-1}$$

Actually, the functions tabulated are the P and Q functions. The extraction of the hyperbolic functions leads to data which are readily interpolable in both z and q. The table must, therefore, be used in conjunction with a table of hyperbolic functions.

There are four basic tables. They provide 7D approximations to $Pc_r(x, q)$, $Qc_r(x, q)$, $Ps_r(x, q)$, and $Qs_r(x, q)$ for $q = 0\ (0.05)\ 1$; $r = 0\ (1)\ 7$, $x = 0\ (0.02)\ 1$, and $r = 8\ (1)\ 15$, $x = 0\ (0.01)\ 1$.

In addition, the values of $M_r(q)$ are furnished to 8S, as are those of the functions $C_r(q)$ and $S_r(q)$, which are defined on pages 1 and 197. The latter can be used instead of $M_r(q)$, corresponding to a different normalization. Also, the eigenvalues $a_r(q)$ and $b_r(q)$ are given to 8D. The computations were performed by a stepwise numerical integration of the differential equations for the P and Q functions. Some of the computations were performed on an 1103 ERA computer; the rest, on an IBM 7090.

The superscript appearing in $Mc_r^{(1)}(z, q)$ indicates that these are functions of the first kind (corresponding to Bessel functions for $q = 0$). A table for functions of the second kind is now in preparation.

Preceding the table is a good general discussion. A helpful chart relates the many non-standardized notations in this field.

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16 [L].—L. S. BARK & P. I. KUZNETSOV, *Tablitsy tsilindricheskikh funktsiĭ ot dvukh mnimykh peremennykh* (*Tables of Cylinder Functions of Two Imaginary Variables*), Computing Center, Acad. Sci. USSR, Moscow, 1962, xx + 265 p., 27 cm. Price 2.87 rubles.

On replacing x and y in the Lommel functions of two variables

$$\sum_{m=0}^{\infty} (-1)^m \left(\frac{y}{x}\right)^{n+2m} J_{n+2m}(x)$$

by ix and iy respectively, one is led to the functions

$$\Upsilon_n(y, x) = \sum_{m=0}^{\infty} \left(\frac{y}{x}\right)^{n+2m} I_{n+2m}(x)$$

which are here tabulated. These functions may thus be regarded as Lommel functions of two pure imaginary variables. A collection of formulas in the volume uses also the closely related notation

$$\theta_n(y, x) = \sum_{m=0}^{\infty} \left(\frac{x}{y}\right)^{n+2m} I_{n+2m}(x).$$

The tables give values of $\Upsilon_1(y, x)$ and $\Upsilon_2(y, x)$ to 7S for $y = 0(.01)1(.1)20$, $x = 0(.01)1(.1)y$. There are also second differences in both x and y. Although these are denoted by Δ_{xx}^2 and Δ_{yy}^2, they are central differences; the second equation of line 2 on page xiv should accordingly read $\Delta_{xx}^2 f(x_0) = f(x_1) - 2f(x_0) + f(x_{-1})$. Ordinary Everett coefficients of second differences are tabulated to 8D without differences at interval 0.001. The scheme of bivariate interpolation recommended is clearly set out, with a diagram on page xiv and worked examples, and will be quite intelligible to anyone who does not read Russian.

These extensive tables (computed on the electronic computing machine STRELA) are a development of part of a small table published by Kuznetsov in 1947; see *MTAC*, v. 3, 1948, p. 186 (*for* Kuznetsev, *read* Kuznetsov), or FMRC *Index*, second edition, 1962, Art. 20.72.

There is mention of several integrals which have been shown by Kuznetsov to be expressible in terms of Lommel functions of two imaginary variables. No fewer than nine fields of application are briefly mentioned; in eight of these cases, the bibliography includes at least one reference in a Western language. The present tables have been made to remedy a lack which has made numerical applications difficult, and are clearly of importance.

A. F.

17 [L].—J. W. McClain, F. C. Schoenig, Jr. & N. J. Palladino, *Table of Bessel Functions to Argument 85*, Engineering Research Bulletin B-85, The Pennsylvania State University, University Park, Pennsylvania, September 1962, v. + 30 p., 28 cm. Price \$1.00.

This table consists of 4S values, in floating-point form, of the Bessel functions $J_n(x)$, $Y_n(x)$, $I_n(x)$, and $K_n(x)$ for $n = 0, 1$ and $x = 0(0.1)85$.

An introduction of five pages describes the conventional mathematical procedures used in the underlying calculations, which were performed on an IBM 7074 system, using a Fortran program reproduced in the Appendix.

One infers from the Preface that the authors were apparently unaware of the existence of such fundamental related tables as those of Harvard Computation Laboratory [1] and of the British Association for the Advancement of Science [2].

Moreover, the reliability of the least significant figure appearing in the table under review is uncertain, as revealed by a comparison with the corresponding entries in the fundamental tables cited. Such examination has disclosed 26 terminal-digit errors in the entire range of values tabulated herein for $J_0(x)$ and $J_1(x)$ and