by $ix$ and $iy$ respectively, one is led to the functions

$$\Upsilon_n(y, x) = \sum_{m=0}^{\infty} \left(\frac{y}{x}\right)^{n+2m} I_{n+2m}(x)$$

which are here tabulated. These functions may thus be regarded as Lommel functions of two pure imaginary variables. A collection of formulas in the volume uses also the closely related notation

$$\theta_n(y, x) = \sum_{m=0}^{\infty} \left(\frac{x}{y}\right)^{n+2m} I_{n+2m}(x).$$

The tables give values of $\Upsilon_1(y, x)$ and $\Upsilon_2(y, x)$ to 7S for $y = 0(.01)1(.1)20$, $x = 0(.01)1(.1)y$. There are also second differences in both $x$ and $y$. Although these are denoted by $\Delta_{xx}^2$ and $\Delta_{yy}^2$, they are central differences; the second equation of line 2 on page xiv should accordingly read $\Delta_{xx}^2 f(x_0) = f(x_1) - 2f(x_0) + f(x_{-1})$. Ordinary Everett coefficients of second differences are tabulated to 8D without differences at interval 0.001. The scheme of bivariate interpolation recommended is clearly set out, with a diagram on page xiv and worked examples, and will be quite intelligible to anyone who does not read Russian.

These extensive tables (computed on the electronic computing machine STRELA) are a development of part of a small table published by Kuznetsov in 1947; see *MTAC*, v. 3, 1948, p. 186 (*for* Kuznetsev, *read* Kuznetsov), or FMRC *Index*, second edition, 1962, Art. 20.72.

There is mention of several integrals which have been shown by Kuznetsov to be expressible in terms of Lommel functions of two imaginary variables. No fewer than nine fields of application are briefly mentioned; in eight of these cases, the bibliography includes at least one reference in a Western language. The present tables have been made to remedy a lack which has made numerical applications difficult, and are clearly of importance.

A. F.

17 [L].—J. W. McClain, F. C. Schoenig, Jr. & N. J. Palladino, *Table of Bessel Functions to Argument 85*, Engineering Research Bulletin B-85, The Pennsylvania State University, University Park, Pennsylvania, September 1962, v. + 30 p., 28 cm. Price $1.00.

This table consists of 4S values, in floating-point form, of the Bessel functions $J_n(x)$, $Y_n(x)$, $I_n(x)$, and $K_n(x)$ for $n = 0, 1$ and $x = 0(0.1)85$.

An introduction of five pages describes the conventional mathematical procedures used in the underlying calculations, which were performed on an IBM 7074 system, using a Fortran program reproduced in the Appendix.

One infers from the Preface that the authors were apparently unaware of the existence of such fundamental related tables as those of Harvard Computation Laboratory [1] and of the British Association for the Advancement of Science [2].

Moreover, the reliability of the least significant figure appearing in the table under review is uncertain, as revealed by a comparison with the corresponding entries in the fundamental tables cited. Such examination has disclosed 26 terminal-digit errors in the entire range of values tabulated herein for $J_0(x)$ and $J_1(x)$ and

27 errors (ranging up to 25 units) in the tabulated values of the remaining functions for $x \leqq 20$.

Thus we have still another example of the result of insufficient planning to insure complete tabular accuracy, especially in the vicinity of zeros of oscillatory functions such as $J_n(x)$ and $Y_n(x)$, where single-precision computations cannot always be relied upon to yield the desired number of significant figures.

Despite these flaws, the present table performs a valuable service in listing approximations to the values of $Y_n(x)$, $I_n(x)$, and $K_n(x)$ over a range not hitherto tabulated.

J. W. W.

1. Harvard University, Computation Laboratory, *Annals*, v. 3, *Tables of the Bessel Function of the First Kind of Orders Zero and One*, Harvard University Press, Cambridge, Massachusetts, 1947.

2. British Association for the Advancement of Science, Committee on Mathematical Tables, *Mathematical Tables*, v. 10, *Bessel Functions, Part II: Functions of Positive Integer Order*, Cambridge University Press, Cambridge, England, 1952.

**18 [I, M].**—Ruel V. Churchill, *Fourier Series and Boundary Value Problems*, McGraw-Hill Book Company, Inc., New York, 1963, viii + 248 p., 23 cm. Price $6.75.

Professor Churchill has written several very successful text books at the intermediate level. Each of these books has been marked by careful attention to mathematical detail. They are extremely easy to teach from and contain plenty of exercises for the student.

This second edition of *Fourier Series and Boundary Value Problems* continues this tradition. The chapter topics are: 1. Partial Differential Equations of Physics; 2. Superposition of Solutions; 3. Orthogonal Sets of Functions; 4. Fourier Series; 5. Further Properties of Fourier Series; 6. Fourier Integrals; 7. Boundary Value Problems; 8. Bessel Functions and Applications; 9. Legendre Polynomials and Applications; and 10. Uniqueness of Solutions.

These chapter headings are, in many ways, similar to those in the first edition. A separate chapter on the Fourier integral has been added, and various chapters have been rearranged. The lists of problems have been either added to or, in many cases, completely replaced with new ones. References have now been placed in a bibliography at the end of the book.

These changes appear to the reviewer to be definite improvements over the first edition. The book should supply the needs for an intermediate text for students in mathematics, physics, or engineering. The only criticism that the reviewer can make concerns the lists of problems. There are plenty of problems, but no really difficult ones. For the most part, they are simple applications of material presented in the text. It would seem that the book would appeal to mathematics students more if it contained some additional, difficult problems dealing with applications and with further mathematical theory.

Teachers will welcome Professor Churchill's book as a clear, well written text which makes their job easier for them.

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