the concept of a norm, an effective notation for particular types of matrices, and the so-called König ratio and its generalizations. The emphasis throughout is on matrix problems, and presumably much of this material will be developed in greater detail in his forthcoming book on matrices.

D. S.

46[K].—A. E. Sarhan & B. G. Greenberg, editors, *Contributions to Order Statistics*, John Wiley & Sons, Inc., New York, 1962, xxv + 482 p., 24 cm. Price $11.25.

If the random observations $X_1, X_2, \cdots x_n$ of a sample drawn from a continuous population are arranged in ascending order of magnitude, $X_{(1)} \leqq X_{(2)} \leqq \cdots \leqq X_{(n)}$, then we have the order statistics of the sample and $X_{(i)}$ is called the ith order statistic. Order statistics are inherently much more informative than the ordinary random sample alone, and therefore have considerable practical value. It is probably for this reason that within the last fifteen years there has occurred a rather large-scale attack on the theory of order statistics.

Interest in order statistics runs high. For example, are the least or greatest values, or both, "outliers" which perhaps should be discarded? What are the distribution properties of the order statistics and how efficient are the order statistics (in particular, various linear combinations of them) in estimating population parameters? The great, practical point regarding order statistics is that computations involved in their use are rather minimal compared to that for the "most efficient" statistics, while the loss in efficiency is not very significant.

The present volume brings together the more pertient theoretical background, applications, and tables required to use order statistics. Indeed, it provides a very worthwhile manual, which is sorely needed at the present state of progress in this area of Mathematical Statistics. As examples of topics covered, we mention in particular the exact and approximate distributions and moments of order statistics from normal, exponential and gamma populations, the range $X_{(n)} - X_{(1)}$, best linear estimates of population parameters, theory and applications of extreme values, tests for suspected outlying observations, the maximum variance ratio for several independent samples, multiple-decision and multiple-comparison techniques for ranking treatment means, optimum grouping and spacing of observations, short-cut tests, and tolerance regions. From this list alone, we get a general idea concerning the over-all value of the book as a welcome addition to the statistical library. The editors of the book are to be congratulated for a job well done.

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47[K, X].—Edward O. Thorp, *Beat the Dealer: A Winning Strategy for the Game of Twenty-One*, Random House, New York, 1962, xiii + 236 p., 21 cm. Price $4.95.

Although volumes have been written about blackjack, the first mathematical attempt to obtain an optimal strategy was made in 1956 by Baldwin, Cantey, Maisch, and McDermott. To simplify the computations, they assumed that all hands were dealt from a complete shuffled deck. As the game is actually played, however, the later hands come from a decreasing deck. Thus, the probability of winning and the optimal strategy should fluctuate. Further, the player should have the advantage frequently. Using an IBM 704, the author computed, as a function of the cards in the depleted deck, the situations when the player has the advantage.

The book begins with a discussion of the rules of the game and then proceeds to describe the optimal strategy as a function of the amount of information (the cards depleted from deck) the player is able to remember. If no information is remembered, the optimal strategy yields 0.21 percent advantage to the casino. However, keeping track of the fives, the player obtains an advantage of 3 percent. If a player is able to keep track of more than four cards, tens and aces, he can obtain an advantage which ranges from 4 to 15 percent.

The book contains an account of the author's successful test in Nevada. The chapter on how to spot cheating is unique. The book also contains an appendix giving the probabilities for hands dealt from a complete deck.

MELVIN DRESHER

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48[L].—ROLIN F. BARRETT, *Tables of Modified Struve Functions of Orders Zero and Unity*, MS of 55 typewritten sheets $8\frac{1}{2}$ x 11 in., deposited in the UMT File.

Following a one-page introduction, which gives the general definition of the Struve functions, their expansions in both power series and asymptotic series, and an outline of the contents of the tables, the author presents decimal approximations to $L_0(x)$ and $L_1(x)$ to 5 and 6S for $x = 0.02(0.005)\,4(0.05)\,10(0.1)\,19.2$, calculated by power series, and approximations to 2S, in floating-point form, for $x = 6(0.25)\,59.50(0.5)\,100$, calculated by asymptotic series. All calculations were performed on an IBM 650 at North Carolina State College, where the author is a member of the Department of Mechanical Engineering.

No bibliography is presented, and apparently no comparison of these data was made with existing tables such as those of the National Bureau of Standards [1]. A single comparison with the latter tables revealed numerous last-place errors (ranging up to 5 units) in the tables under review.

Apart from these discrepancies, the manuscript tables appear to be reliable, and they supply tabular information corresponding to a range of the argument extending considerably beyond that of previous tables of these functions.

J. W. W.

1. National Bureau of Standards, *Tables of Functions and of Zeros of Functions*, Applied Mathematics Series, v. 37, U. S. Government Printing Office, Washington, D. C., 1954, p. 113–119.

49[L].—AVNER FRIEDMAN, *Generalized Functions and Partial Differential Equations*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1963, xii + 340 p., 23 cm. Price $10.00.

The main subject of this book by Avner Friedman is a somewhat specialized topic in the theory of partial differential equations; namely, the Cauchy problem