

Evaluation at Half Periods of Weierstrass' Elliptic Function with Rhombic Primitive Period-Parallelogram

By Chih-Bing Ling and Chen-Peng Tsai

In a recent note [1], the senior author evaluated the Weierstrass' elliptic function at half periods to 16D for the case of rectangular primitive period-parallelogram, i.e., the three functions

$$(1) \quad e_1 = \wp(\omega_1), \quad e_2 = \wp(\omega_2), \quad e_3 = \wp(\omega_3)$$

with double periods $2\omega_1 = 1$ and $2\omega_2 = ai$; ω_3 being defined by

$$(2) \quad \omega_1 + \omega_2 + \omega_3 = 0.$$

In this note, these three functions will be evaluated, also to 16D, for the case of rhombic primitive period-parallelogram. Two methods of evaluation are given. One is similar to that described previously [1] and the other is in terms of Jacobian Theta functions. As no generality is lost, here the double periods are represented by $2\omega_1 = 1$ and $2\omega_2 = \frac{1}{2} + ci$, where c is a positive real quantity.

It is known that these three functions form a set of distinct roots of the following cubic [2],

$$(3) \quad x^3 - (15\sigma_4)x - (35\sigma_6) = 0$$

where, in the case under consideration,

$$(4) \quad \begin{aligned} \sigma_{2k} &= \sum'_{m,n=-\infty}^{\infty} \frac{1}{(2m\omega_1 + 2n\omega_2)^{2k}} \\ &= 2 \sum_{m=1}^{\infty} \frac{1}{m^{2k}} + 2 \sum_{n=1}^{\infty} \sum_{m=-\infty}^{\infty} \frac{1}{\{m + n(\frac{1}{2} + ci)\}^{2k}} \\ &= 2 \sum_{m=1}^{\infty} \frac{1}{m^{2k}} + 2 \sum_{n=1}^{\infty} \sum_{m=-\infty}^{\infty} \frac{1}{\{m + \frac{1}{2} + (2n - 1)ci\}^{2k}} \\ &\quad + 2 \sum_{n=1}^{\infty} \sum_{m=-\infty}^{\infty} \frac{1}{(m + 2n ci)^{2k}}. \end{aligned}$$

The accent on the first summation sign denotes the omission of simultaneous zeros of m and n . The cubic (3) indicates that the sum of the three functions vanishes identically, i.e.,

$$(5) \quad e_1 + e_2 + e_3 = 0.$$

In a similar manner as described previously [1], it can be shown that

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$$\begin{aligned}
 & \frac{1}{\pi^4} \sum_{m=-\infty}^{\infty} \frac{1}{(m+ix)^4} = \frac{2}{3 \sinh^2 \pi x} + \frac{1}{\sinh^4 \pi x}, \\
 & \frac{1}{\pi^6} \sum_{m=-\infty}^{\infty} \frac{1}{(m+ix)^6} = -\frac{2}{15 \sinh^2 \pi x} - \frac{1}{\sinh^4 \pi x} - \frac{1}{\sinh^6 \pi x}, \\
 (6) \quad & \frac{1}{\pi^4} \sum_{m=-\infty}^{\infty} \frac{1}{(m+\frac{1}{2}+ix)^4} = -\frac{2}{3 \cosh^2 \pi x} + \frac{1}{\cosh^4 \pi x}, \\
 & \frac{1}{\pi^6} \sum_{m=-\infty}^{\infty} \frac{1}{(m+\frac{1}{2}+ix)^6} = \frac{2}{15 \cosh^2 \pi x} - \frac{1}{\cosh^4 \pi x} + \frac{1}{\cosh^6 \pi x}.
 \end{aligned}$$

Consequently, we have

$$(7) \quad \sigma_4 = \frac{4\pi^4}{3} \left(\frac{1}{60} + K_1 + K_3 \right), \quad \sigma_6 = \frac{4\pi^6}{15} \left(\frac{1}{126} - K_2 + K_4 \right)$$

where

$$\begin{aligned}
 (8) \quad K_1 &= \sum_{n=2,4,6,\dots}^{\infty} \left(\frac{1}{\sinh^2 n\pi c} + \frac{3}{2 \sinh^4 n\pi c} \right), \\
 K_2 &= \sum_{n=2,4,6,\dots}^{\infty} \left(\frac{1}{\sinh^2 n\pi c} + \frac{15}{2 \sinh^4 n\pi c} + \frac{15}{2 \sinh^6 n\pi c} \right), \\
 K_3 &= \sum_{n=1,3,5,\dots}^{\infty} \left(\frac{1}{\cosh^2 n\pi c} - \frac{3}{2 \cosh^4 n\pi c} \right), \\
 K_4 &= \sum_{n=1,3,5,\dots}^{\infty} \left(\frac{1}{\cosh^2 n\pi c} - \frac{15}{2 \cosh^4 n\pi c} + \frac{15}{2 \cosh^6 n\pi c} \right).
 \end{aligned}$$

The evaluation of e_1 , e_2 , and e_3 requires the solution of the cubic (3). The real root gives the value of e_1 and the pair of complex conjugate roots the values of e_2 and e_3 . The former can be found from the cubic (3) directly by using Newton's method and the latter from the depressed equation of the cubic.

A second method of evaluation may be as follows. It is known that [3]*

$$\begin{aligned}
 (9) \quad 12\omega_1^2 e_1 &= \pi^2(\theta_3^4 + \theta_4^4), \\
 12\omega_1^2 e_2 &= -\pi^2(\theta_2^4 + \theta_3^4), \\
 12\omega_1^2 e_3 &= \pi^2(\theta_2^4 - \theta_4^4)
 \end{aligned}$$

where θ_2 , θ_3 , and θ_4 are Jacobian Theta functions defined by

$$\begin{aligned}
 (10) \quad \theta_2 &= 2q^{1/4} \sum_{n=0}^{\infty} q^{n(n+1)}, \\
 \theta_3 &= 1 + 2 \sum_{n=1}^{\infty} q^{n^2}, \\
 \theta_4 &= 1 + 2 \sum_{n=1}^{\infty} (-1)^n q^{n^2}
 \end{aligned}$$

in which

* e_2 and e_3 have been permuted to conform with the present notation. ω_1 is identified with ω in [3].

$$(11) \quad q = \exp(i\pi\omega_2/\omega_1).$$

It is noted that the preceding three Theta functions are not all independent but are connected by the relation

$$(12) \quad \theta_2^4 + \theta_4^4 = \theta_3^4.$$

Now denote

$$(13) \quad \begin{aligned} p &= e^{-\pi c}, \\ A &= \frac{1}{2} + p^4 + p^{16} + p^{36} + \dots, \\ B &= p + p^9 + p^{25} + \dots. \end{aligned}$$

We then find, for $\omega_2/\omega_1 = \frac{1}{2} + ci$,

$$(14) \quad \begin{aligned} e_1 &= \frac{32\pi^2}{3} (A^4 - 6A^2B^2 + B^4), \\ e_2 &= -\frac{1}{2}e_1 - 64\pi^2(A^3B - AB^3)i, \\ e_3 &= -\frac{1}{2}e_1 + 64\pi^2(A^3B - AB^3)i. \end{aligned}$$

TABLE 1

c	σ_4	σ_6
0.05	2.16464 64674 10478 $\times 10^4$	-2.03468 61239 92188 $\times 10^6$
0.1	1.35285 51098 10968 $\times 10^3$	-3.17943 85391 87710 $\times 10^4$
0.15	2.65424 44894 89441 $\times 10^2$	-2.83086 37451 64231 $\times 10^3$
0.2	7.67059 68640 15799 $\times 10$	-5.92699 26236 38137 $\times 10^2$
0.25	1.93710 87114 46967 $\times 10$	-2.45332 24175 29611 $\times 10^2$
$\sqrt{3}/6$	0	-1.58301 85572 24858 $\times 10^2$
0.3	-3.62453 58222 15253	-1.40825 71800 11392 $\times 10^2$
0.35	-1.29338 25507 01956 $\times 10$	-8.18148 86969 71514 $\times 10$
0.4	-1.55320 57313 46534 $\times 10$	-4.14353 39381 59347 $\times 10$
0.45	-1.47955 79359 61305 $\times 10$	-1.52034 34879 06628 $\times 10$
0.5	-1.26048 48008 61559 $\times 10$	0
0.6	-7.49518 25513 33490	1.04695 23857 33236 $\times 10$
0.7	-3.54385 93824 64415	9.96973 81357 38993
0.8	-1.04685 36509 89181	7.37508 83157 10668
$\sqrt{3}/2$	0	5.86303 16934 25402
0.9	4.02787 83609 98395 $\times 10^{-1}$	5.22023 89750 45370
1	1.21069 29446 54355	3.83331 62773 90017
1.25	1.96367 27971 88045	2.42769 61786 42181
1.5	2.12275 22872 98572	2.11722 17565 99152
1.75	2.15593 25105 11680	2.05187 98601 55075
2	2.16283 47970 89782	2.03826 19185 60751
2.5	2.16456 81756 97548	2.03484 06650 80135
3	2.16464 30841 25693	2.03469 28023 27625
4	2.16464 64611 04163	2.03468 61364 40353
5	2.16464 64674 10478	2.03468 61239 92188
6	2.16464 64674 22254	2.03468 61239 68942
∞	2.16464 64674 22276	2.03468 61239 68898

TABLE 2

c	e_1	e_2 and e_3
0.05	-6.57973 62673 89319 × 10 ²	3.28986 81336 94660 × 10 ²
0.1	-1.64492 81173 83390 × 10 ²	8.22464 05869 16948 × 10
0.15	-7.30584 93638 31184 × 10	3.65292 46819 15592 × 10
0.2	-4.07403 58971 91193 × 10	2.03701 79485 95597 × 10
0.25	-2.51415 52474 67755 × 10	1.25707 76237 33877 × 10
$\sqrt{3}/6$	-1.76950 31908 45431 × 10	8.84751 59542 27155
0.3	-1.59549 03411 11651 × 10	7.97745 17055 58256
0.35	-9.84355 86738 94376	4.92177 93369 47188
0.4	-5.50761 80318 03648	2.75380 90159 01824
0.45	-2.33992 65029 49420	1.16996 32514 74710
0.5	0	0
0.6	3.01540 56900 68455	-1.50770 28450 34228
0.7	4.66025 66846 27599	-2.33012 83423 13799
0.8	5.55021 87470 32100	-2.77510 93735 16050
$\sqrt{3}/2$	5.89834 39694 84770	-2.94917 19847 42385
0.9	6.02887 68932 08147	-3.01443 84466 04073
1	6.28538 81186 69387	-3.14269 40593 34693
1.25	6.51845 74355 05910	-3.25922 87177 52955
1.5	6.56699 37384 39408	-3.28349 68692 19704
1.75	6.57708 71865 96458	-3.28854 35932 98229
2	6.57918 55702 81884	-3.28959 27851 40942
2.5	6.57971 24695 33558	-3.28985 62347 66779
3	6.57973 52389 94009	-3.28986 76194 97004
4	6.57973 62654 72430	-3.28986 81327 36215
5	6.57973 62673 89319	-3.28986 81336 94660
6	6.57973 62673 92899	-3.28986 81336 96450
∞	6.57973 62673 92906	-3.28986 81336 96453
		i 1.18989 31466 96521 × 10 ⁻³
		i 7.66281 94807 82929 × 10 ⁻¹
		i 4.66807 08317 54428
		i 9.70788 89192 14831
		i 1.35464 73301 58955 × 10
		i 1.53243 47153 49767 × 10
		i 1.56616 48842 85086 × 10
		i 1.63303 12860 75360 × 10
		i 1.59915 99419 38787 × 10
		i 1.50346 31103 67526 × 10
		i 1.37503 71636 04075 × 10
		i 1.09200 38547 40336 × 10
		i 8.33344 97665 68968
		i 6.22948 64049 74943
		i 5.10811 57178 32557
		i 4.60637 60054 51221
		i 3.38661 83253 97387
		i 1.55326 22270 74373 × 10 ⁻¹
		i 7.09063 29833 67643 × 10 ⁻¹
		i 3.23371 67067 21421 × 10 ⁻¹
		i 1.47445 31124 21317 × 10 ⁻¹
		i 3.06512 77923 13172 × 10 ⁻²
		i 6.37177 83442 38741 × 10 ⁻³
		i 2.75349 51572 21704 × 10 ⁻⁴
		i 1.18989 31466 96490 × 10 ⁻⁵
		i 5.14199 45183 96954 × 10 ⁻⁷
		i 0

σ_4 and σ_6 may be found from the relations

$$(15) \quad \begin{aligned} \sigma_4 &= \frac{1}{3^0}(e_1^2 + e_2^2 + e_3^2), \\ \sigma_6 &= \frac{1}{3^5}e_1e_2e_3. \end{aligned}$$

Values of σ_4 and σ_6 are computed by the first method and those of e_1, e_2, e_3 by the second method for $c = 0.05(0.05)0.5(0.1)1(0.25)2(0.5)3(1)6$ and ∞ . They are then checked by (15). A separate check is also made on the imaginary part of e_2 or e_3 , of which the numerical value is alternately equal to $8\pi^2 pC^4$ by virtue of (12), where

$$(16) \quad C = 1 - p^2 - p^6 + p^{12} + p^{20} - p^{30} - \dots$$

Those for $c = \sqrt{3}/6$ and $c = \sqrt{3}/2$ are also computed. In both such cases, the value of σ_4 vanishes identically. The results to 16D are shown in Tables 1-2. In performing the computation, auxiliary tables of $e^{\pi a}$ and $e^{-\pi a}$ with adequate guarding figures are first prepared for $a = 0.1(0.1)10$. Besides, the following values are prepared.

$$(17) \quad \begin{aligned} e^{\sqrt{3}\pi/2} &= 1.51909 \ 37703 \ 74778 \ 01395 \times 10, \\ e^{-\sqrt{3}\pi/2} &= 6.58287 \ 21011 \ 29665 \ 13439 \times 10^{-2}. \end{aligned}$$

It is thought that the tables of $e^{\pi a}$ and $e^{-\pi a}$ may be of use elsewhere and therefore they are also shown here in Table 3 to 21D.

If σ_{2k} and e_s are expressed as functions of c , the following relations hold.

$$(18) \quad \begin{aligned} \sigma_4(c') &= (2c)^4 \sigma_4(c), \\ \sigma_6(c') &= -(2c)^6 \sigma_6(c) \end{aligned}$$

and

$$(19) \quad \begin{aligned} e_1(c') &= -(2c)^2 e_1(c), \\ e_2(c') &= -(2c)^2 e_3(c), \\ e_3(c') &= -(2c)^2 e_2(c) \end{aligned}$$

where

$$(20) \quad cc' = \frac{1}{4}.$$

It is noted that here the pair of double periods may alternately be represented by the pair $be^{-i\alpha}$ and $be^{i\alpha}$, in which

$$(21) \quad b = (\frac{1}{4} + c^2)^{1/2}, \quad \alpha = \tan^{-1} 2c.$$

It appears that the rectangular and rhombic primitive period-parallelograms are the only cases in which σ_4 and σ_6 are both real.

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TABLE 3

a	$e^{\pi a}$				$e^{-\pi a}$					
0.1	1.36910	77706	24846	88484	7.30402	69104	86456	10873	(-1)*	
0.2	1.87445	60875	85338	35057	5.33488	09109	11032	51176	(-1)	
0.3	2.56633	23952	08135	32094	3.89661	13737	53467	94902	(-1)	
0.4	3.51358	56242	85733	63770	2.84609	54333	60292	80116	(-1)	
0.5	4.81047	73809	65351	65547	2.07879	57635	07619	08547	(-1)	
0.6	6.58606	19626	94724	85836	1.51835	80198	06488	86883	(-1)	
0.7	9.01702	86109	42078	24233	1.10901	27836	41952	22245	(-1)	
0.8	1.23452	83939	18736	85795	(1)	8.10025	92157	94312	88095	(-2)
0.9	1.69020	24171	71154	60199	(1)	5.91645	11294	07757	88852	(-2)
1.0	2.31406	92632	77926	90057	(1)	4.32139	18263	77224	97744	(-2)
1.1	3.16821	02101	17924	35952	(1)	3.15635	62190	61546	64985	(-2)
1.2	4.33762	12176	45429	13719	(1)	2.30541	10763	10682	04408	(-2)
1.3	5.93867	09151	05567	24216	(1)	1.68387	84541	10676	64716	(-2)
1.4	8.13068	04970	54802	50287	(1)	1.22990	93542	81271	53095	(-2)
1.5	1.11317	77848	98562	26027	(2)	8.98329	10211	29427	88966	(-3)
1.6	1.52406	03553	91575	92353	(2)	6.56141	99363	06069	66927	(-3)
1.7	2.08660	28754	67872	35481	(2)	4.79247	87785	78186	16706	(-3)
1.8	2.85678	42110	11213	73297	(2)	3.50043	93966	67033	38810	(-3)
1.9	3.91124	54622	93824	99384	(2)	2.55673	03551	78298	63043	(-3)
2.0	5.35491	65552	47647	36503	(2)	1.86744	27317	07988	81443	(-3)
2.1	7.33145	78668	37191	20979	(2)	1.36398	51966	18748	94871	(-3)
2.2	1.00375	55935	49546	24197	(3)	9.96258	45816	08502	26303	(-4)
2.3	1.37424	95829	36839	19562	(3)	7.27669	85882	06595	17424	(-4)
2.4	1.88149	57827	76781	53285	(3)	5.31492	02307	75977	42717	(-4)
2.5	2.57597	04965	97570	55092	(3)	3.88203	20392	67662	47233	(-4)
2.6	3.52678	12237	92079	54462	(3)	2.83544	66482	18162	15904	(-4)
2.7	4.82854	35787	87543	23018	(3)	2.07101	78621	83408	03034	(-4)
2.8	6.61079	65345	18733	02916	(3)	1.51267	70197	48574	28994	(-4)
2.9	9.05089	29054	29406	22166	(3)	1.10486	33659	11803	90247	(-4)
3.0	1.23916	47807	91669	74815	(4)	8.06995	17570	30459	92392	(-5)
3.1	1.69655	01304	66510	05654	(4)	5.89431	44799	67793	83033	(-5)
3.2	2.32275	99668	76296	70719	(4)	4.30522	31580	55474	73556	(-5)
3.3	3.18010	87199	46649	78016	(4)	3.14454	65802	08667	28460	(-5)
3.4	4.35391	15599	10793	22659	(4)	2.29678	52843	12226	31551	(-5)
3.5	5.96097	41492	87215	58845	(4)	1.67757	81524	22578	70825	(-5)
3.6	8.16121	60282	82962	95159	(4)	1.22530	75969	73866	47167	(-5)
3.7	1.11735	84282	07025	47474	(5)	8.94967	96619	20613	64147	(-6)
3.8	1.52978	41066	31403	67916	(5)	6.53687	01090	90149	06637	(-6)
3.9	2.09443	93077	67444	13665	(5)	4.77454	75187	14898	47932	(-6)
4.0	2.86751	31313	66532	99747	(5)	3.48734	23562	08995	49178	(-6)
4.1	3.92593	45105	22707	69216	(5)	2.54716	42415	82974	76484	(-6)
4.2	5.37502	74453	20893	81174	(5)	1.86045	56165	95087	22855	(-6)
4.3	7.35899	18427	10655	01612	(5)	1.35888	17889	37618	96908	(-6)
4.4	1.00752	52915	82001	87728	(6)	9.92530	91545	70345	60877	(-7)
4.5	1.37941	07058	05983	40218	(6)	7.24947	25159	87938	10837	(-7)

TABLE 3—Continued

a	$e^{\pi a}$					$e^{-\pi a}$				
4.6	1.88856	19162	02076	47085	(6)	5.29503	42343	60785	53694	(-7)
4.7	2.58564	47947	78413	81653	(6)	3.86750	72539	71822	59597	(-7)
4.8	3.54002	63806	06813	87980	(6)	2.82483	77059	51176	91520	(-7)
4.9	4.84667	76259	05740	65394	(6)	2.06326	90622	02422	28930	(-7)
5.0	6.63562	39993	41134	23327	(6)	1.50701	72753	90064	61075	(-7)
5.1	9.08488	43804	42670	74431	(6)	1.10072	94734	01701	04231	(-7)
5.2	1.24381	85800	49235	82598	(7)	8.03975	76948	91610	22921	(-8)
5.3	1.70292	16831	92971	91696	(7)	5.87226	06557	27887	98907	(-8)
5.4	2.33148	33092	25041	56976	(7)	4.28911	49854	82733	65777	(-8)
5.5	3.19205	19157	42137	17455	(7)	3.13278	11276	13661	21624	(-8)
5.6	4.37026	30820	80489	01787	(7)	2.28819	17660	75428	61239	(-8)
5.7	5.98336	11453	51290	55253	(7)	1.67130	14235	76846	05372	(-8)
5.8	8.19186	62385	55235	84751	(7)	1.22072	30573	33960	68163	(-8)
5.9	1.12155	47723	12530	90732	(8)	8.91619	40610	18549	86382	(-9)
6.0	1.53552	93539	54466	93923	(8)	6.51241	21360	79900	72821	(-9)
6.1	2.10230	51705	21611	64610	(8)	4.75668	33494	10617	94951	(-9)
6.2	2.87828	23451	85932	29437	(8)	3.47429	43188	75800	38156	(-9)
6.3	3.94067	87248	46367	75598	(8)	2.53763	39200	01905	86296	(-9)
6.4	5.39521	38637	23174	97636	(8)	1.85349	46440	65715	51964	(-9)
6.5	7.38662	92250	06302	56497	(8)	1.35379	74758	69850	17560	(-9)
6.6	1.01130	91470	68071	94014	(9)	9.88817	31951	02024	39235	(-10)
6.7	1.38459	12117	54883	38231	(9)	7.22234	83112	57602	86121	(-10)
6.8	1.89565	45871	52483	67960	(9)	5.27522	26422	33194	26958	(-10)
6.9	2.59535	54256	91101	44405	(9)	3.85303	68137	67871	77194	(-10)
7.0	3.55332	12808	47044	35970	(9)	2.81426	84574	85552	72109	(-10)
7.1	4.86487	97771	34322	34756	(9)	2.05554	92546	80768	60938	(-10)
7.2	6.66054	47060	30274	03357	(9)	1.50137	87072	01871	18741	(-10)
7.3	9.11900	35136	20234	64547	(9)	1.09661	10480	23383	27988	(-10)
7.4	1.24848	98570	85274	50206	(10)	8.00967	66050	99546	92873	(-11)
7.5	1.70931	71648	81753	87131	(10)	5.85028	93467	94089	01097	(-11)
7.6	2.34023	94129	02041	86211	(10)	4.27306	70823	11625	73556	(-11)
7.7	3.20403	99653	26715	06920	(10)	3.12105	96959	51795	89636	(-11)
7.8	4.38667	60139	21370	58104	(10)	2.27963	04008	46659	38446	(-11)
7.9	6.00583	22178	73377	47364	(10)	1.66504	81793	74702	70619	(-11)
8.0	8.22263	15585	59499	52750	(10)	1.21615	56709	40930	83974	(-11)
8.1	1.12576	68761	80890	65249	(11)	8.88283	37478	93270	22920	(-12)
8.2	1.54129	61780	91317	24348	(11)	6.48804	56735	98971	03015	(-12)
8.3	2.11020	05742	59200	32308	(11)	4.73888	60196	43211	04041	(-12)
8.4	2.88909	20037	95285	41152	(11)	3.46129	51013	20206	20908	(-12)
8.5	3.95547	83124	46234	88485	(11)	2.52813	92565	17773	08240	(-12)
8.6	5.41547	60941	08196	18967	(11)	1.84655	97163	06303	62722	(-12)
8.7	7.41437	04020	76625	99127	(11)	1.34873	21859	72147	77499	(-12)
8.8	1.01510	72131	77397	90288	(12)	9.85117	61813	79790	86996	(-13)
8.9	1.38979	11735	78508	43714	(12)	7.19532	55928	74119	82759	(-13)
9.0	1.90277	38952	92161	29169	(12)	5.25548	51760	06448	55522	(-13)

TABLE 3—Continued

a	$e^{\pi a}$					$e^{-\pi a}$				
9.1	2.60510	25257	86606	78556	(12)	3.83862	05153	21374	94491	(-13)
9.2	3.56666	61113	28858	90997	(12)	2.80373	87543	05271	02946	(-13)
9.3	4.88315	02882	44645	96778	(12)	2.04785	83311	41947	37983	(-13)
9.4	6.68555	90047	64705	70119	(12)	1.49576	12359	52466	78871	(-13)
9.5	9.15325	07843	94276	31478	(12)	1.09250	80319	05929	90960	(-13)
9.6	1.25317	86775	39217	86849	(13)	7.97970	80649	63507	85426	(-14)
9.7	1.71573	66654	00312	45627	(13)	5.82840	02444	31926	72695	(-14)
9.8	2.34902	84009	45530	65488	(13)	4.25707	92230	41663	13798	(-14)
9.9	3.21607	30371	52984	44509	(13)	3.10938	21205	16908	18044	(-14)
10.0	4.40315	05860	63202	90114	(13)	2.27110	10683	24093	83868	(-14)

* (n) at the end of the number stands for the factor 10^n .

Institute of Mathematics, Academia Sinica
Aeronautical Research Laboratory
Taiwan, China

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