If we again choose $a = -b = 2$, the second transformation becomes

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 1 & 0 \\ 0 & 0 & \frac{2}{3} & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 2 & 0 & 0 & 0 \\ -1 & -2 & 2 & 0 & 0 \\ 0 & -6 & 7 & 2 & 0 \\ 0 & 2 & 0 & 2 & 1 \\ 0 & 4 & -2 & 1 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{1}{3} & 1 & 0 \\ 0 & 0 & -\frac{2}{3} & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 2 & 0 & 0 & 0 \\ -1 & -2 & 2 & 0 & 0 \\ 0 & -6 & \frac{19}{3} & 2 & 0 \\ 0 & 0 & \frac{7}{9} & \frac{8}{3} & 1 \\ 0 & 0 & -\frac{1}{9} & \frac{7}{3} & 3 \end{bmatrix}.$$

In the matrix on the right we see that

$$S_3 \neq 0,$$

and we can carry out the third step. At the completion of the third step, the matrix will be in tridiagonal form.

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1. C. D. La Budde, "The reduction of an arbitrary real square matrix to tri-diagonal form using similarity transformations," *Math. Comp.*, v. 17, 1963, p. 433–437.

A Note on La Budde's Algorithm

By Beresford Parlett

In the October 1963 issue of *Mathematics of Computation*, La Budde presented an algorithm for the reduction of an arbitrary real square matrix A to a similar tridiagonal matrix. We show here that when applied to Hessenberg matrices this procedure is identical to the more familiar reduction by elimination methods. Therefore the same care is needed with the new technique as with elimination in treating the instabilities which can occur, see [1] and [3].

Let A be an unreduced lower Hessenberg matrix; i.e., $a_{ij} = 0$ if $j > i + 1$, $a_{i,i+1} \neq 0$. La Budde's algorithm [2] consists of a sequence of major steps at the jth of which the current matrix A is transformed to

$$A' = V_j(I_{n-j} + axy^t)AV_j(I_{n-j} + bxy^t)$$

using the notation of [2]. The equations (3.8)–(3.11) determining the vectors x, y and the scalar $c^{-1} = \sum_{k=j+1}^{n} x_k a_{jk}$ reduce, in this case, to

$$c^{-1} = -(a + b)a_{j,j+1}a_{j+1,j}/ab, \tag{3.8}$$

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$$x_{j+1} = c^{-1}a_{j,j+1}^{-1}, \tag{3.9}$$

$$y_{j+1} = a_{j+1,j}^{-1}, \tag{3.10}$$

$$x_k = -a_{kj}/a, \qquad y_k = 0, \qquad k = j+2, \cdots, n. \tag{3.11}$$

Now let $D(q)$ be I_{n-j} except for the $(1, 1)$ element which is q, let $e^t = (1, 0, \cdots, 0)$, and $m^t = (0, m_{j+2,j}, \cdots, m_{nj})$ where $m_{kj} = a_{kj}/a_{j+1,j}$. Then

$$I_{n-j} + axy^t = D(-a/b)(I_{n-j} - me^t),$$

$$I_{n-j} + bxy^t = (I_{n-j} + me^t)D(-b/a).$$

However $V_j(I_{n-j} - me^t)AV_j(I_{n-j} + me^t)$ represents the jth step of the reduction of A to tridiagonal form by elimination, see [1] and [3]. The matrices $D(-a/b)$ and $D(-b/a)$ represent the multiplication of row $j + 1$ and the division of column $j + 1$ by $-a/b$ and this leaves $S'_{j+1} = a'_{j+2,j+1}a'_{j+1,j+2}$ invariant for all permissible a, b. In general a, b are meant to be chosen so that $S'_{j+1} \neq 0$ and thus in this case they are nugatory and we may put $a = b = 1$.

This shows that the penultimate paragraph of p. 436 in [2] is not correct. For Hessenberg matrices the algorithm can break down.

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A Note on Projective Planes of Order Nine

By E. T. Parker and R. B. Killgrove

Veblen and Wedderburn [1] in 1907 constructed three non-Desarguesian projective planes of order nine, two duals of one another and one self-dual plane. These planes, together with the Desarguesian, constitute the only projective planes of order nine discovered to date; it has, however, not been proved that no more exist.

Hall, Swift, and Killgrove [2], using the SWAC and card sorter, determined all permutation representations of planes of order nine, under the restriction that a set of nine parallel permutations forms the noncyclic group. No new planes were found; but, interestingly, more than one such coordinatization (i.e., nonisomorphic ternary rings with elementary abelian addition) was obtained for two known non-Desarguesian planes. Killgrove [3] carried out the same search for the cyclic group in place of the elementary abelian, obtaining no plane.

Among the latin squares in the coordinatizations of planes in [2], exactly five are nonisomorphic, exclusive of the group. This determination was facilitated con-

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