There have been several effects on the theory of the Riemann zeta function as a result of this computation. First of all, zeros of $\zeta'(s)$ were discovered in the critical strip. The evidence of this calculation and further calculations results in the following conjecture: There are zeros of $\zeta'(s)$ on dense set of vertical lines for $\frac{1}{2} < \sigma \leqq 1$, and no zeros of $\zeta'(s)$ for $\sigma \leqq \frac{1}{2}$ except on the negative real axis. A paper is being prepared to show that $\zeta'(s) \neq 0$ in a very large portion of the left half plane $(\sigma \leqq \sigma_0, t \geqq t_0)$.

A second result is that $|\zeta(1 - s)| > |\zeta(s)|$ for $\frac{1}{2} < \sigma \leqq 1$, $t \geqq 10$, if $\zeta(s) \neq 0$. Thus, always for this region $|\zeta(1 - s)| \geqq |\zeta(s)|$, and the strictness of this inequality is equivalent to the Riemann hypothesis.

AUTHOR'S SUMMARY

1. A. FLETCHER, J. C. P. MILLER, L. ROSENHEAD & L. J. COMRIE, *An Index of Mathematical Tables*, Addison-Wesley, Reading, Massachusetts, 1962.

2. J. I. HUTCHINSON, "On the roots of the Riemann Zeta Function," *Trans. Amer. Math. Soc.*, v. 27, 1925, p. 49–60.

79[L].—(a) MARIYA I. ZHURINA & LENA N. KARMAZINA, *Tablitsy funktsii Lezhandra* $P_{-1/2+i\tau}(x)$, Tom II (*Tables of the Legendre functions* $P_{-1/2+i\tau}(x)$, Vol. II), Akad. Nauk SSSR, Moscow, 1962, iv + 414 p., 27 cm. Price 4.42 rubles.

(b) M. I. ZHURINA & L. N. KARMAZINA, *Tablitsy i formuly dlya sfericheskikh funktsii* $P^m_{-1/2+i\tau}(z)$ (*Tables and formulas for the spherical functions* $P^m_{-1/2+i\tau}(z)$), Akad. Nauk SSSR, Moscow, 1962, xivii + 58 p., 27 cm. Price 0.58 rubles.

Both volumes are in the series of *Mathematical Tables* of the Computational Centre of the Academy of Sciences of the USSR.

(a) This second volume, promised in the first volume and mentioned in the review of that volume (*Math. Comp.*, v. 16, p. 253–254, April 1962, where *for* Karamazina *read* Karmazina and *for* Izdatel'stov *read* Izdatel'stvo), has now been published. It will be remembered that Vol. I was for $x^2 < 1$ and that Vol. II was to be for $x > 1$. Replacing $-\frac{1}{2} + i\tau$ by s for convenience in the whole of the present reviews, the second volume does indeed give values of $P_s(x)$ to 7D without differences for $\tau = 0(0.01)50$ and $x = 1.1(0.1)2(0.2)5(0.5)10(10)60$. The main table occupies pages 11–270, i–iv, 271–407, a total of 401 pages. In principle there are four pages for each of the hundred ranges of width 0.50 in τ, but the table for $\tau = 32.50(0.01)33.00$ occupies five pages, the material having been skillfully and hardly noticeably spaced out (presumably to retrieve an error in pagination).

On pages 408–413 is an auxiliary table which for $x = 1.01(0.01)3(0.05)5(0.1)10$ gives, to 7D without differences, values of $\theta = \cosh^{-1}x$ and of the first four coefficients in the expansion of $P_s(\cosh\theta)$ in multiples of $\tau^{-n}J_n(\tau\theta)$. The values of θ have been read against the Harvard 9D tables [1], and appear to be correct on the convention that rounding is always downward, except that upward rounding occurs at $x = 1.61, 1.68, 1.72, 1.83, 2.00, 4.45, 7.60$. Nine decimals are not enough to decide at $x = 2.04$, but special calculation shows that upward rounding occurs here also.

(b) In this slim volume, which relates to both $x^2 < 1$ and $x > 1$, the same authors give first, on pages v–xxxviii, a collection of formulas relating to $P_s{}^m(z)$. Then follow a description of the tables and a bibliography of 43 items.

The eight tables on pages 1–56 fall into two groups.

Tables 1 and 2 list for $x = -0.99(0.01) + 0.99$, to 7D, $\theta = \cos^{-1} x$ and coefficients for the calculation of $P_s(\cos\theta)$ and $P_s^1(\cos\theta)$ when $I_0(\tau\theta)$ and $I_1(\tau\theta)$ are known, while Tables 3 and 4 list for $x = 1.01(0.01)3(0.05)5(0.1)10(1)60$, to 7D, $\eta = \cosh^{-1} x$ and coefficients for the calculation of $P_s(\cosh\eta)$ and $P_s^1(\cosh\eta)$ when $J_0(\tau\eta)$ and $J_1(\tau\eta)$ are known.

Tables 5 to 8 do not require values of Bessel functions to be available. Tables 5 and 6 list for $x = -0.90(0.01) + 0.99$, to 7D, values of $\theta = \cos^{-1} x$ and the first eight coefficients in the expansions of $P_s(\cos\theta)$ and $(1 + 4\tau^2)^{-1} P_s^1(\cos\theta)$ in powers of τ^2. Tables 7 and 8 list for $x = 1.01(0.01)3(0.05)5(0.1)10(1)60$, to 7D, $\eta = \cosh^{-1} x$ and the first eight coefficients in the expansions of $P_s(\cosh\eta)$ and $(1 + 4\tau^2)^{-1} P_s^1(\cosh\eta)$ in powers of τ^2.

There are no differences. Roundings in $\cosh^{-1} x$ for $x \leqq 10$ are as in (a) above, while for $10 < x \leqq 60$ there are upward roundings at $x = 35$ and 59, and unfortunately a major error at $x = 11$, where final 689 should be 699.

Taking the three volumes as a whole, the authors have achieved a gratifying fullness of coverage.

A. F.

1. Harvard University, *Annals of the Computation Laboratory*, v. 20, *Tables of Inverse Hyperbolic Functions*, Harvard University Press, Cambridge, Massachusetts, 1949.

80[L, M].—K. Singh, J. F. Lumley & R. Betchov, *Modified Hankel Functions and their Integrals to Argument 10*, Engineering Research Bulletin B-87, The Pennsylvania State University, University Park, Pennsylvania, October 1963, v + 29 p., 28 cm. Price $1.00.

Let

$$h_1(z) = (12)^{1/6} e^{-i\pi/6}[Ai(-z) - iBi(-z)] = (\tfrac{2}{3} z^{3/2})^{1/3} H_{1/3}^{(1)}(\tfrac{2}{3} z^{3/2}),$$

$$h_2(z) = (12)^{1/6} e^{i\pi/6}[Ai(-z) + iBi(-z)] = (\tfrac{2}{3} z^{3/2})^{1/3} H_{1/3}^{(2)}(\tfrac{2}{3} z^{3/2})$$

where the usual notation for Airy functions and Hankel functions is used. Tables are presented for the real and imaginary parts of

$$h(z), \int_0^s h(iu)\,du, \int_0^s \int_0^v h(iu)\,du\,dv, z = is,$$

for $s = -10(0.1)10$, where h stands for h_1 or h_2. The number of significant figures varies from 8 to 4. Most of the tables are new, though there is some overlap with the tables of M. V. Cerrillo and W. H. Kautz (see *Math. Comp.*, v. 16, 1962, p. 390). The functions were computed using ascending series and asymptotic series representations. The latter are not given in the text. For these and other representations, see Y. L. Luke, *Integrals of Bessel Functions*, McGraw-Hill Book Co., 1963. I find it most irritating that this report containing work sponsored by the U. S. government should carry a price tag. This petty practice should be discontinued.

Y. L. L.