

## REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

90[G, X].—J. H. WILKINSON, *Rounding Errors in Algebraic Processes*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1964, vi + 161 p., 23 cm. Price \$6.00.

The numerical solution of a mathematical problem, if not algebraic in form to begin with, requires at some point a reduction to algebraic form, implicitly or explicitly. The errors involved in the reduction are truncation errors, and these can be bounded by methods that are fairly standard and classical. The need for establishing rigorous bounds for the rounding errors that arise in the numerical evaluation of algebraic expressions, and in solving algebraic equations, became acute only with the advent of the electronic computer, and the techniques for doing so have developed only within the past two decades. In fact, the first systematic study to appear in the open literature was in the now classical paper by von Neumann and Goldstine, published in 1947, dealing with the inversion of positive-definite matrices by fixed-point computation. This paper led to a curious and fortuitous misconception on the part of many readers, that the positioning for size, which played so prominent a role, was essential for the control of error. It is indeed essential for the control of errors in the inversion of matrices that are not positive definite. For inverting matrices that are positive definite, it is essential only to permit the computations to be done in fixed point, and it has no bearing on the numerical stability of the process.

Probably no one has had more extensive practical experience than the author of this volume in computations of the type therein described and assessed, and no one has contributed more to an understanding of the subject. The appearance of the volume is, therefore, an event of major importance in the history of computing. There are three chapters. The first is on the fundamental arithmetic operations, both fixed-point and floating-point; the second on computations involving polynomials; and the third on matrix computations. This last includes both inversion and the evaluation of characteristic roots and vectors. Much progress has been made since the appearance of the paper by von Neumann and Goldstine. In particular, whereas in that paper it was suggested that the problem of solving  $Ax = h$ , with  $A$  other than positive definite, be replaced by  $A^T Ax = A^T h$ , since the problem of finding error bounds seemed otherwise intractable, it turns out that the problem is in fact entirely manageable, and even fairly simple, when properly viewed.

This is not to imply that all the problems are trivial, or even completely solved. Faster machines and bigger memories will demand sharper results and more refined techniques. But much of the mystery is removed, and no practicing computationist can afford not to have this book within easy reach.

A. S. H.

91[H, M, W, X].—THOMAS L. SAATY & JOSEPH BRAM, *Nonlinear Mathematics*, McGraw-Hill Book Company, Inc., New York, 1964, xv + 381 p., 23 cm. Price \$12.50.

This book consists of six chapters on various parts of analysis of contemporary interest where nonlinear functions and functionals enter in an essential fashion: linear and nonlinear transformations; nonlinear algebraic and transcendental func-

tions; nonlinear optimization, nonlinear programming and systems of inequalities, nonlinear ordinary differential equations; introduction to automatic control; and linear and nonlinear prediction theory.

Of these chapters, only that on nonlinear optimization may be considered to be well enough organized and to contain enough material to represent a contribution to mathematical literature. The others show lack of understanding of the basic ideas and methods, lack of organization, or both. This is particularly true of the chapters on control theory and nonlinear differential equations.

The book is definitely not recommended for either students or teachers.

RICHARD BELLMAN

The RAND Corporation  
Santa Monica, California

**92[K, O, X, Z].**—JOHN PEMBERTON, *How to Find Out in Mathematics (A Guide to Sources of Mathematical Information Arranged According to the Dewey Decimal Classification)*, Macmillan, New York, 1963, x + 158 p., 19 cm. Price \$2.45 (paperbound).

This is a useful guide, not to the substance of mathematics, but more to its organizational set-up. It is written from the point of view of the librarian. The list of titles of the twelve chapters and three appendices that follows should indicate its scope:

Careers for Mathematicians; The Organization of Mathematical Information; Mathematical Dictionaries, Encyclopedias and Theses; Mathematical Periodicals and Abstracts; Mathematical Societies; Mathematical Education; Computers and Mathematical Tables; Mathematical History and Biography; Mathematical Books—Part 1: Bibliographies; Mathematical Books—Part 2: Evaluation and Acquisition; Probability and Statistics; Operational Research and Related Techniques; Sources of Russian Mathematical Information; Mathematics and the Government; Actuarial Science.

D. S.

**93[I, M].**—V. M. BELIĀKOV, R. I. KRAVŤSOVA & M. G. RAPPAPORT, *Tablitsy elliptičeskih integralov*, Tom I (*Tables of Elliptic Integrals*, v. I), Izdatel'stvo Akademii Nauk SSSR, Moscow, 1962, 656 p., 27 cm. Price 5 rubles 14 kopecks.

This is the third set of extensive tables of the elliptic integral of the third kind to appear within the last five years. The previous ones were prepared, respectively, by Selfridge and Maxfield [1] and by Paxton and Rollin [2].

In the present member of a two-volume set we find in Table I the values of

$$\Pi(n, k^2, \varphi) = \int_0^\varphi (1 + n \sin^2 \alpha)^{-1} (1 - k^2 \sin^2 \alpha)^{-1/2} d\alpha$$

to 7S for  $-n = 0(0.05)0.85, 0.88(0.02)(0.94)(0.01)0.98(0.005)1$ ,  $k^2 = 0(0.01)1$ , and  $\varphi = 0^\circ(1^\circ)90^\circ$ . Corresponding to  $n = 0$ ,  $\Pi(n, k^2, \varphi)$  reduces, of course, to  $F(k^2, \varphi)$ .

Table II gives  $E(k^2, \varphi)$  to similar precision for the same range in  $k^2$  and  $\varphi$ .

Approximations to 8D of  $A_m(\varphi) = \int_0^\varphi \sin^{2m} \alpha d\alpha$  appear in Table III for  $m = 1(1)10$