where $\dot{+}$ denotes addition modulo one. Thus (3) is equivalent to requiring that $\Theta_1 \dot{+} \Theta_2$ and Θ_1 have the same distribution. If we set

$$\phi(n) = E\{e^{2\pi i n \Theta_1}\} = \int_0^1 e^{2\pi i n \theta_1}\, dF_{\Theta_1}(\theta_1),$$

then (3) and the independence of Θ_1, Θ_2 imply

$$\phi(n) = E\{e^{2\pi i n(\Theta_1 \dot{+} \Theta_2)}\} = E\{e^{2\pi i n(\Theta_1 + \Theta_2)}\} = \phi^2(n)$$

so that $\phi(n) = 0$ or 1. Certainly $\phi(0) = 1$. There are two cases to be examined.

Case 1. $\phi(n) = 0$ for all $n \neq 0$.

It follows from the uniqueness theorem for Fourier-Stieltjes series that $dF_{\Theta_1}(d\theta_1) = d\theta_1$ and hence $\Pr(M_1 \leqq x) = F_\infty(x)$.

Case 2. $\phi(n) = 1$ for some $n \neq 0$.

Let m be the smallest positive integer such that $\phi(m) = 1$. Then

$$0 = \int_0^1 (1 - e^{2\pi i m \theta_1})\, dF_{\Theta_1}(\theta_1) = \int_0^1 (1 - \cos 2\pi m \theta_1)\, dF_{\Theta_1}(\theta_1).$$

It follows that F_{Θ_1} is a 'step function' with points of discontinuity at $\theta_k = k/m$ $(k = 1, 2, \cdots, m)$ and, hence, $\phi(n + m) = \phi(n)$ $(n = 0, \pm 1, \pm 2, \cdots)$. We assert that $\phi(n) = 1$ if and only if $n = km$ for some integer k; for if $\phi(n) = 1$ with $km < n < (k + 1)m$ then $\phi(n - km) = \phi(n) = 1$ while $0 < n - km < m$ contradicting the minimality of m. The uniqueness theorem for Fourier-Stieltjes series now shows that $\Pr(M_1 \leqq x) = F_m(x)$.

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New Primes of the Form $n^4 + 1$

By A. Gloden

This note presents some results of a continuation of the author's systematic factorization of integers of the form $n^4 + 1$ [1].

An electronic computer at l'Institut Blaise Pascal in Paris has been used to find solutions of the congruence $x^4 + 1 \equiv 0 \pmod{p}$ for all primes of the form $8k + 1$ in the interval $10^6 < p < 4 \cdot 10^6$, thereby extending the previous range of such tables listed in [1].

With the aid of these tables, the complete factorization of $n^4 + 1$ has now been effected for all even values of n less than 2040 and for all odd values less than 2397.

Thus, the primality of $\frac{1}{2}(n^4 + 1)$ has been established for the following 116 values of n:

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1007	1163	1401	1655	1853	2051	2205	2349
1013	1173	1441	1683	1865	2061	2209	2363
1015	1183	1457	1687	1905	2069	2215	2369
1019	1253	1463	1737	1909	2071	2223	2373
1041	1259	1483	1745	1915	2073	2245	
1047	1269	1485	1751	1935	2079	2247	
1049	1275	1493	1755	1945	2097	2255	
1053	1305	1527	1757	1967	2125	2261	
1057	1327	1529	1765	1977	2131	2279	
1071	1333	1533	1789	1985	2141	2283	
1087	1353	1547	1809	2001	2143	2305	
1101	1355	1557	1813	2007	2145	2311	
1119	1371	1567	1823	2011	2149	2315	
1123	1381	1569	1829	2013	2163	2333	
1125	1383	1571	1841	2037	2175	2341	
1135	1389	1635	1849	2039	2193	2343	

Similarly, corresponding to the following 94 values of n, the integer $n^4 + 1$ has been shown to be prime:

1038	1170	1322	1472	1598	1688	1824	1942
1042	1180	1330	1486	1610	1700	1836	1944
1072	1200	1344	1496	1612	1706	1850	1948
1076	1202	1382	1536	1618	1710	1854	1952
1088	1218	1388	1540	1622	1718	1864	1956
1126	1236	1404	1542	1638	1722	1870	1962
1132	1238	1406	1552	1644	1738	1892	1972
1136	1246	1428	1554	1646	1754	1910	1978
1142	1252	1434	1558	1650	1772	1916	1986
1144	1270	1442	1568	1652	1788	1926	1994
1150	1280	1446	1586	1666	1806	1932	
1152	1302	1458	1594	1680	1820	1934	

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1. A. Gloden, "Additions to Cunningham's Factor Table of $n^4 + 1$," *Math. Comp.*, v. 16, 1962, p. 239–241.

Some Additional Factorizations of $2^n \pm 1$

By K. R. Isemonger

Herein are set forth some details of three new factorizations of integers of the form $2^n \pm 1$.

The first of these is the complete factorization of $2^{119} - 1$, which possesses as algebraic factors the Mersenne primes $2^7 - 1 = 127$ and $2^{17} - 1 = 131071$. The quotient is known to be divisible by 239 and 20231. There then remains the factorization of the integer

$$N = 82\ 57410\ 95583\ 43357\ 90279,$$

which was proved composite by E. Gabard of Poitiers, France.

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